Fasci di iperpiani.

Iperpiano di $\mathbb{R}^n$ = sottospazio affine $\boxed{(n-1)\text{-dimensionale}}$ di $\mathbb{R}^n$.

$$\sum_{i=1}^{n} a_i x_i + d = 0$$

$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n + d = 0$$

$$\Downarrow \text{ in } \mathbb{R}^3$$

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + d = 0$$

$$ax + by + cz + d = 0$$

1) $\boxed{2x+y-3z+1}=0$

$\cancel{a=0}$

2) $\boxed{x-y+5z+7}=0$

$$\boxed{a\,\cancel{(2x+y-3z+1)}+b\,\cancel{(x-y+5z+7)}=0}$$

$$\underbrace{\boxed{(2a+b)}}_{A}x+\underbrace{\boxed{(a-b)}}_{B}y+\underbrace{\boxed{(-3a+5b)}}_{C}z+\underbrace{\boxed{(a+7b)}}_{D}=0$$

Fascio di piani per la retta

$$r: \begin{cases} \underline{ax+by+cz+d=0} \\ \underline{a'x+b'y+c'z+d'=0} \end{cases}$$

$$\alpha(ax+by+cz+d)+\beta(a'x+b'y+c'z+d')=0$$

al variare di α e β.

$$\begin{cases} \boxed{x = 1} \\ \boxed{y = -1} \\ \boxed{z = 3} \end{cases}$$

$$x - 1 = 0$$

$$y + 1 = 0$$

$$z - 3 = 0$$

$$\alpha(x-1) + \beta(y+1) + \gamma(z-3) = 0$$

Esercizio: trovare un'eq. cartesiana del piano π che contiene la retta

$$r: \begin{cases} 2x+y-3z+1=0 \\ x-y+5z+7=0 \end{cases}$$

e passa per il punto $(1,1,2)$.

$$\boxed{(2a+b)}x+\boxed{(a-b)}y+\boxed{(-3a+5b)}z+\boxed{(a+7b)}=0$$

$$(2a+b)1+(a-b)1+(-3a+5b)2+(a+7b)=0$$

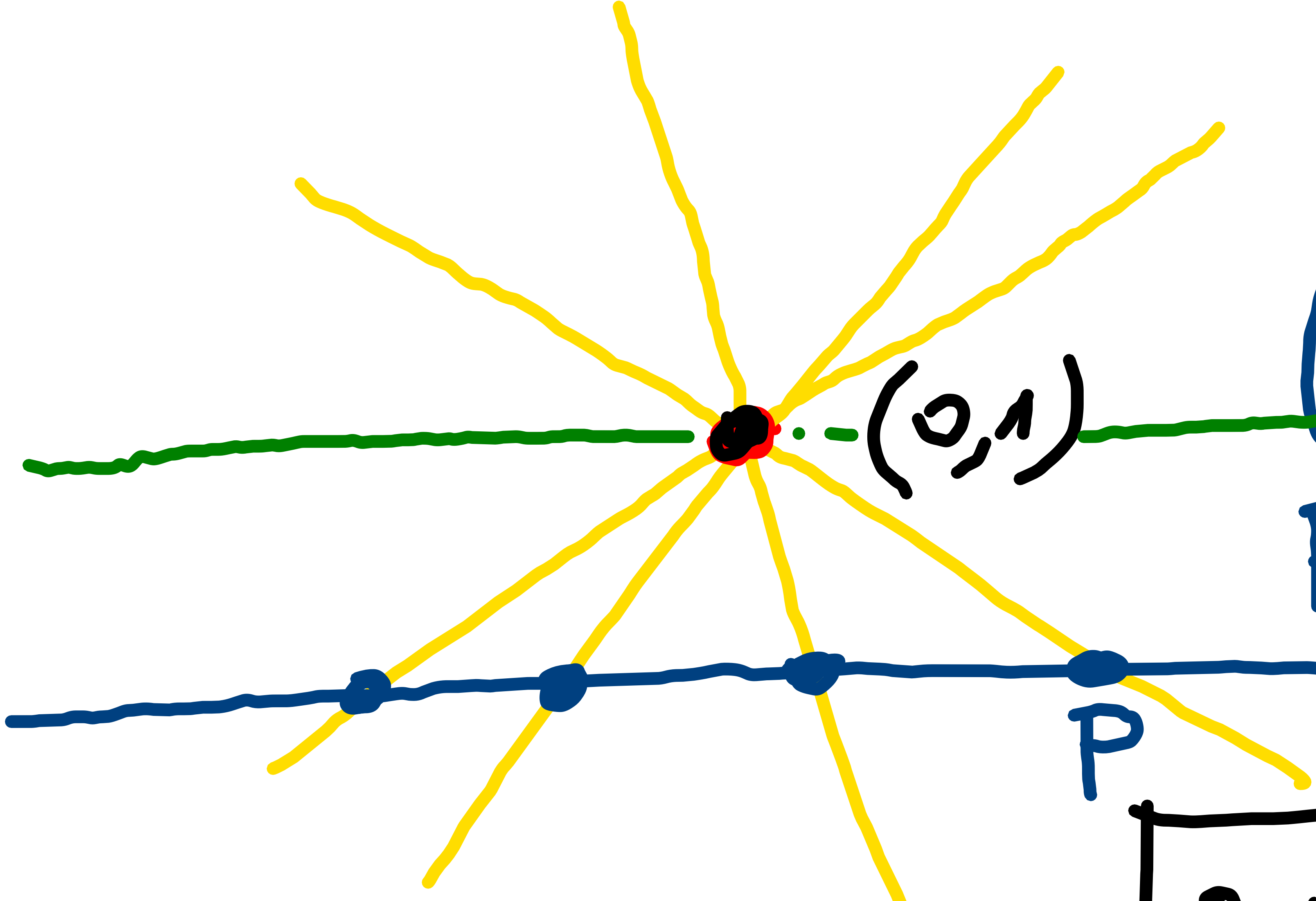
$$-2a+17b=0$$

$$a=\frac{17}{2}b \qquad (a,b)=(17,2)$$

$$36x+15y-41z+31=0$$

$$\begin{cases} a_1x_1 + \dots + a_nx_n + d = c \\ a_1'x_1 + \dots + a_n'x_n + d' = 0 \end{cases}$$

$$C = \left(\begin{array}{ccc|c} a_1 & \cdots & a_n & -d \\ a_1' & \cdots & a_n' & -d' \end{array}\right)$$



$$\begin{cases} ax+by-b=0 \\ y=0 \end{cases}$$

$P=\left(\frac{b}{a},0\right)$

$y=0$

$x=0$

$y=1$

$x=0$

$y-1=0$

$a(x)+b(y-1)=0$

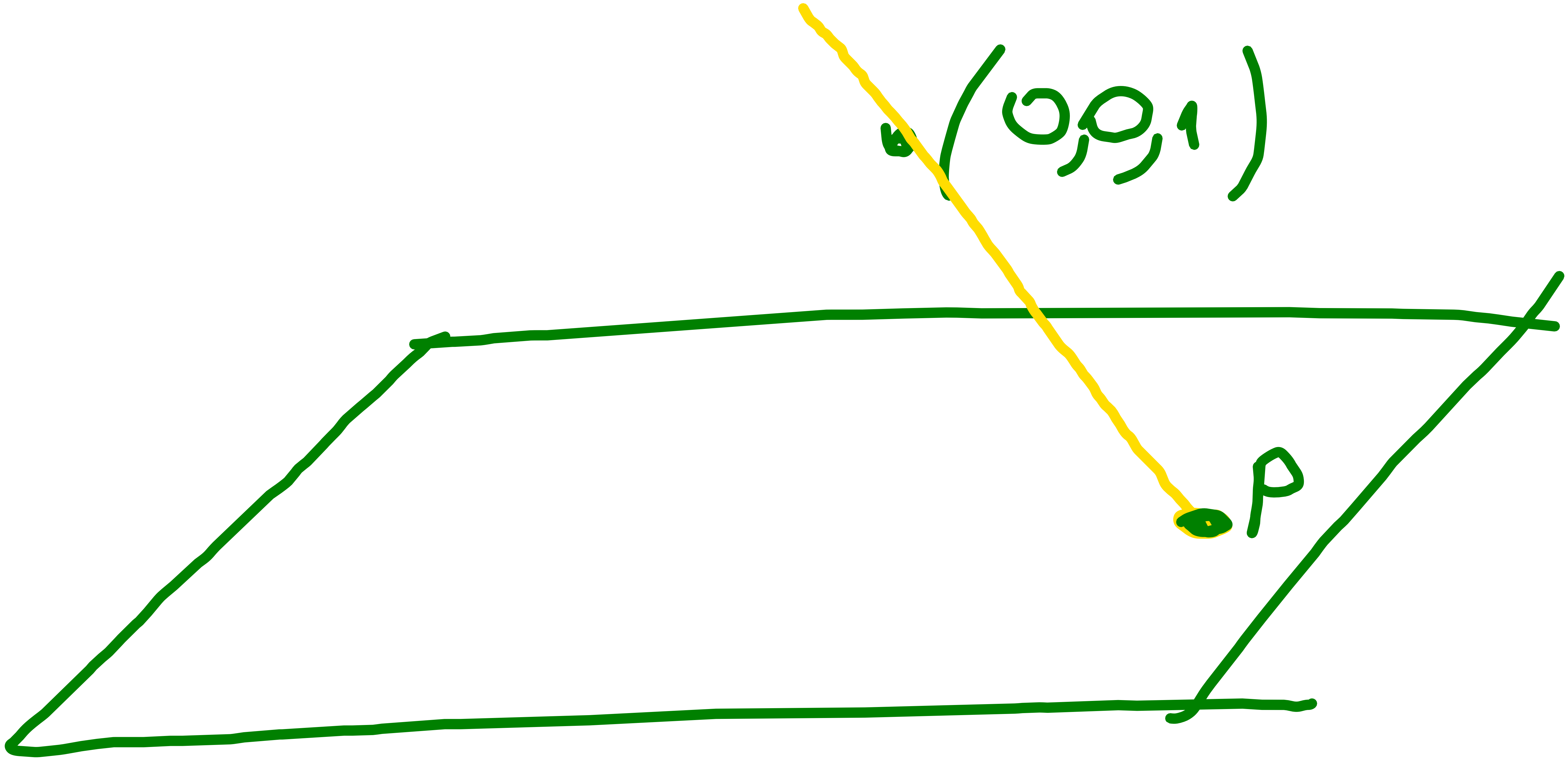
$ax+by-b=0$

$\Downarrow a=0$

$by=b$

$\Downarrow$

$y=1$


w (0,0,1)
P

$(\mathbb{Z}, +, \cdot)$ $\square$ $\bigcirc$

Anello $(A, +, \cdot)$

$+ : A \times A \to A$

$\cdot : A \times A \to A$

1) $(A, +)$ è un gruppo commutativo.

2) $a \cdot (b+c) = a \cdot b + a \cdot c$
$(a+b) \cdot c = a \cdot c + b \cdot c$

3) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

$(M_n(\mathbb{R}), +, \cdot)$ è un anello

prodotto righe per colonne

1) $(M_n(\mathbb{R}), +)$ è un gruppo comm.

2) $(A+B)\cdot C = A\cdot C + B\cdot C$

$A\cdot(B+C) = A\cdot B + A\cdot C$

3) $A\cdot(B\cdot C) = (A\cdot B)\cdot C$

$$\begin{pmatrix} 2 & -8 \\ 10 & 22 \end{pmatrix}$$

$$Q + 0 = 0 + Q = Q$$

$$Q \cdot 1 = 1 \cdot Q = Q$$

Come verificare se una forma bilineare simmetrica è definita positiva.

$$(x_1, x_2)\begin{pmatrix}1 & 2\\ 2 & 8\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

$$f((x_1,x_2),(x_1,x_2)) \geq 0$$

$$\det A > 0$$

$$\mathrm{Tr}\,A > 0$$

$$X A {}^tY \quad \left(X=(x_1,\dots,x_n),\ Y=(y_1,\dots,y_n)\right)$$

è una forma bilineare simm. definita positiva se e solo se A è simmetrica e tutti i suoi autovalori sono positivi.

$$(x_1, x_2, x_3)\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$p_A(t) = \det(A - tI)$$

$$\det\begin{pmatrix} 2-t & 0 & 1 \\ 0 & 2-t & 1 \\ 1 & 1 & 3-t \end{pmatrix} = (2-t)(2-t)(3-t) - (2-t) - (2-t)$$

$$= (2-t)\left[(2-t)(3-t) - 2\right]$$

$$(2-t)[(2-t)(3-t)-2] =$$

$$= (2-t)[t^2-5t+4]$$

0

$$\Delta = 25-16 = 9$$

$$t = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$$

autovalori:

$2, 4, 1 > 0$

4

$$\det(A - tI) = 0$$

$$U_t := \{ v \in V : f(v) = tv \}$$

$$U_2 = \langle (-1, 1, 0) \rangle$$

U_2:

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} z \\ z \\ x+y+z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} z = 0 \\ z = 0 \\ x+y+z = 0 \end{cases} \Rightarrow \begin{cases} z = 0 \\ x+y+z = 0 \end{cases} \Rightarrow \begin{cases} x = -s \\ y = s \\ z = 0 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccc} y & y & z \end{array} \\ \begin{pmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 1 & 1 & -1 \end{pmatrix} \end{array} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2x + z = 0 \\ -2y + z = 0 \\ \cancel{x + y - z = 0} \end{cases} \qquad \begin{cases} x = \frac{1}{2}s \\ y = \frac{1}{2}s \\ z = s \end{cases}$$

$$U_4 = \langle (1/2, 1/2, 1) \rangle = \langle (1, 1, 2) \rangle$$

$$\begin{array}{ccc} x & y & z \end{array}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x + z = 0 \\ y + z = 0 \\ \cancel{x + y + 2z = 0} \end{cases} \qquad \begin{cases} x = -s \\ y = -s \\ z = s \end{cases}$$

$$U_1 = \langle (-1, -1, 1) \rangle = \langle (1, 1, -1) \rangle$$

$U_2 = \langle (-1,1,0) \rangle$

$U_4 = \langle (1,1,2) \rangle$

$U_1 = \langle (1,1,-1) \rangle$

$B = ((-1,1,0),(1,1,2),(1,1,-1))$

è una base di $\mathbb{R}^3$ fatta di autovettori dell'endomorfismo considerato.

Base di autovettori (base spettrale)

$$D^2 : C^2(\mathbb{R},\mathbb{R}) \to C^2(\mathbb{R},\mathbb{R})$$

$$D^2(\sin x) = (-1)\sin x$$

$$D^2(x^4) = 12x^2$$

$$D^2(e^{kx}) = k^2 e^{kx}$$

$f: \mathbb{R}^3 \to \mathbb{R}^3$

Base spettrale: (v_1, v_2, v_3)

$$\bar{A}_f = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$f(v_1) = \lambda_1 v_1$

$f(v_2) = \lambda_2 v_2$

$f(v_3) = \lambda_3 v_3$

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$$\begin{array}{cc} & j \\ i & \left(\cdots\cdots\; p_{ij} \;\cdots\cdots\right) \end{array} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$P = \lim_{n\to\infty} A^n X$$