$$+\left(X_0^2+5X_1^2+7X_2^2\right)=0$$

$$11X_0^2+8X_1^2+3X_2^2=0$$

A discriminante di $[f]$ $\quad \overline{P}\equiv(\overline{X}_0,\overline{X}_1,\ldots,\overline{X}_n)$

Un punto $Q\equiv(X_0,\ldots,X_n)$ è coniugato a $\overline{P}$

sse $\quad (X_0\ldots X_n)\cdot \underbrace{A}_{(n+1)\times(n+1)}\cdot \underbrace{\begin{pmatrix}\overline{X}_0\\ \vdots\\ \overline{X}_n\end{pmatrix}}_{(n+1)\times 1}=0$

$$\begin{pmatrix}b_0\\ b_1\\ \vdots\\ b_n\end{pmatrix}$$

Se $\overline{P}\notin V[f]$, allora $\begin{pmatrix}b_0\\ \vdots\\ b_n\end{pmatrix}\neq\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$

se $\overline{P}\in V[f]$, allora $\begin{pmatrix}b_0\\ \vdots\\ b_n\end{pmatrix}=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$

quindi OGNI Q è coniugato a $\overline{P}$

$\overline{P} \equiv (\overline{z}_0, \dots, \overline{z}_n) \quad \overline{Q} \equiv (\overline{y}_0, \dots, \overline{y}_n)$

$\overline{P} \in \tau(\overline{Q}) \iff (\overline{z}_0, \dots, \overline{z}_n)$ saddisfano l'eq. di $\tau(\overline{Q})$: $\iff$

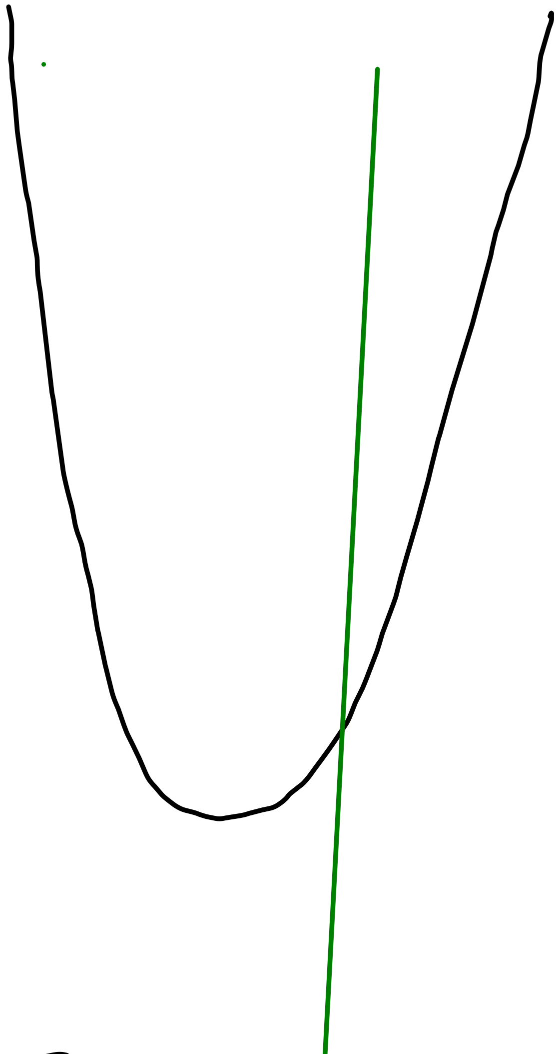
$$(x_0 \dots x_n) \cdot A \cdot \begin{pmatrix} \overline{y}_0 \\ \vdots \\ \overline{y}_n \end{pmatrix} = 0$$

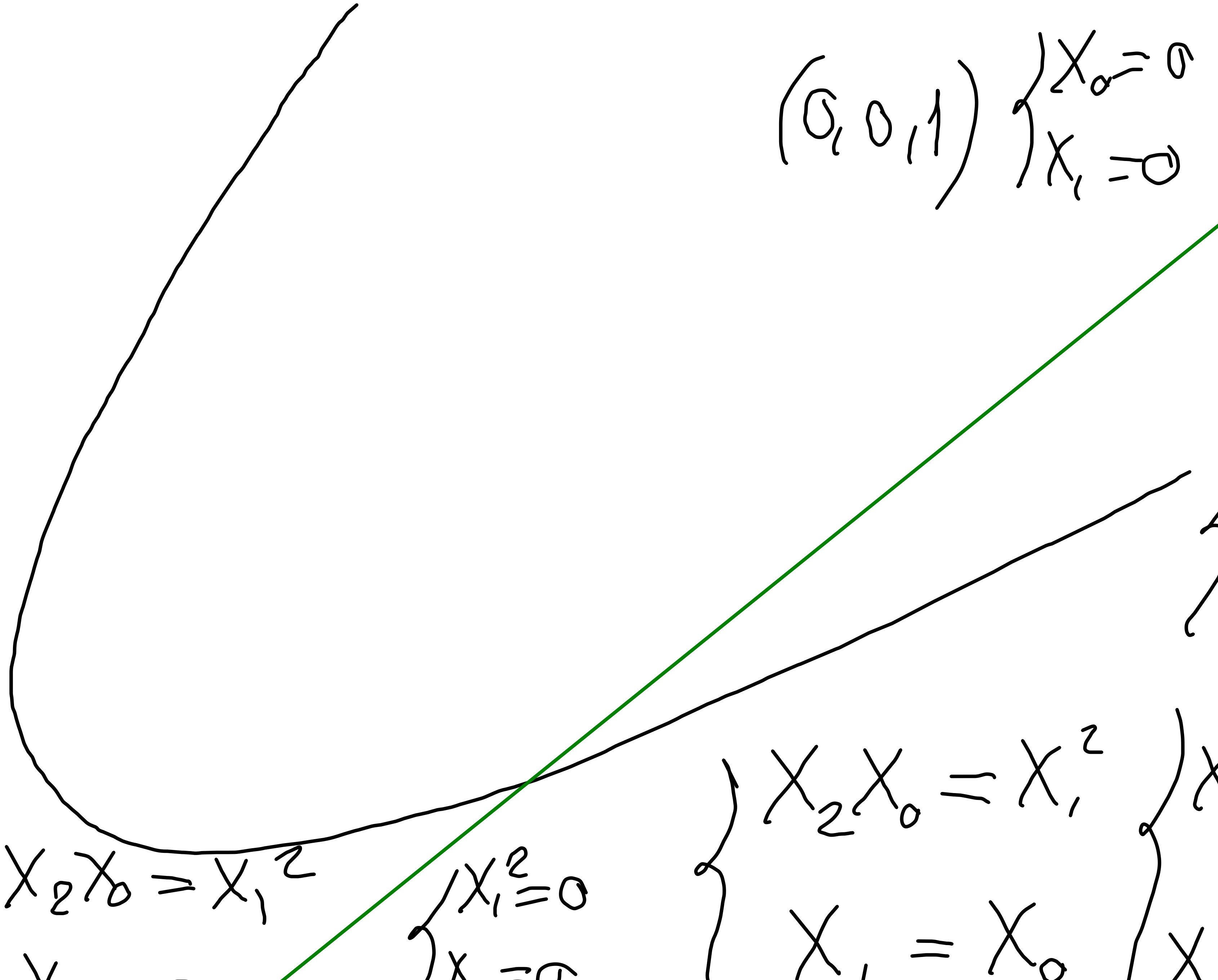
$$\iff (\overline{z}_0 \dots \overline{z}_n) \cdot A \cdot \begin{pmatrix} \overline{y}_0 \\ \vdots \\ \overline{y}_n \end{pmatrix} = 0 \iff$$

$$\iff (\overline{y}_0 \dots \overline{y}_n) \cdot A \cdot \begin{pmatrix} \overline{z}_0 \\ \vdots \\ \overline{z}_n \end{pmatrix} = 0 \iff$$

$\iff (\overline{y}_0, \dots, \overline{y}_n)$ saddisfano l'eq. di $\tau(\overline{P})$: $\iff \overline{Q} \in \tau(\overline{P})$

$$(x_0 \dots x_n) \cdot A \cdot \begin{pmatrix} \overline{z}_0 \\ \vdots \\ \overline{z}_n \end{pmatrix} = 0$$





$(0,0,1)$ $\begin{cases} X_0 = 0 \\ X_1 = 0 \end{cases}$

$\begin{cases} X_2 = X_0 \\ X_1 = X_0 \end{cases}$ $(1,1,1)$ $(1,1)$

$\begin{cases} X_0(X_2 - X_0) = 0 \\ X_1 = X_0 \end{cases}$

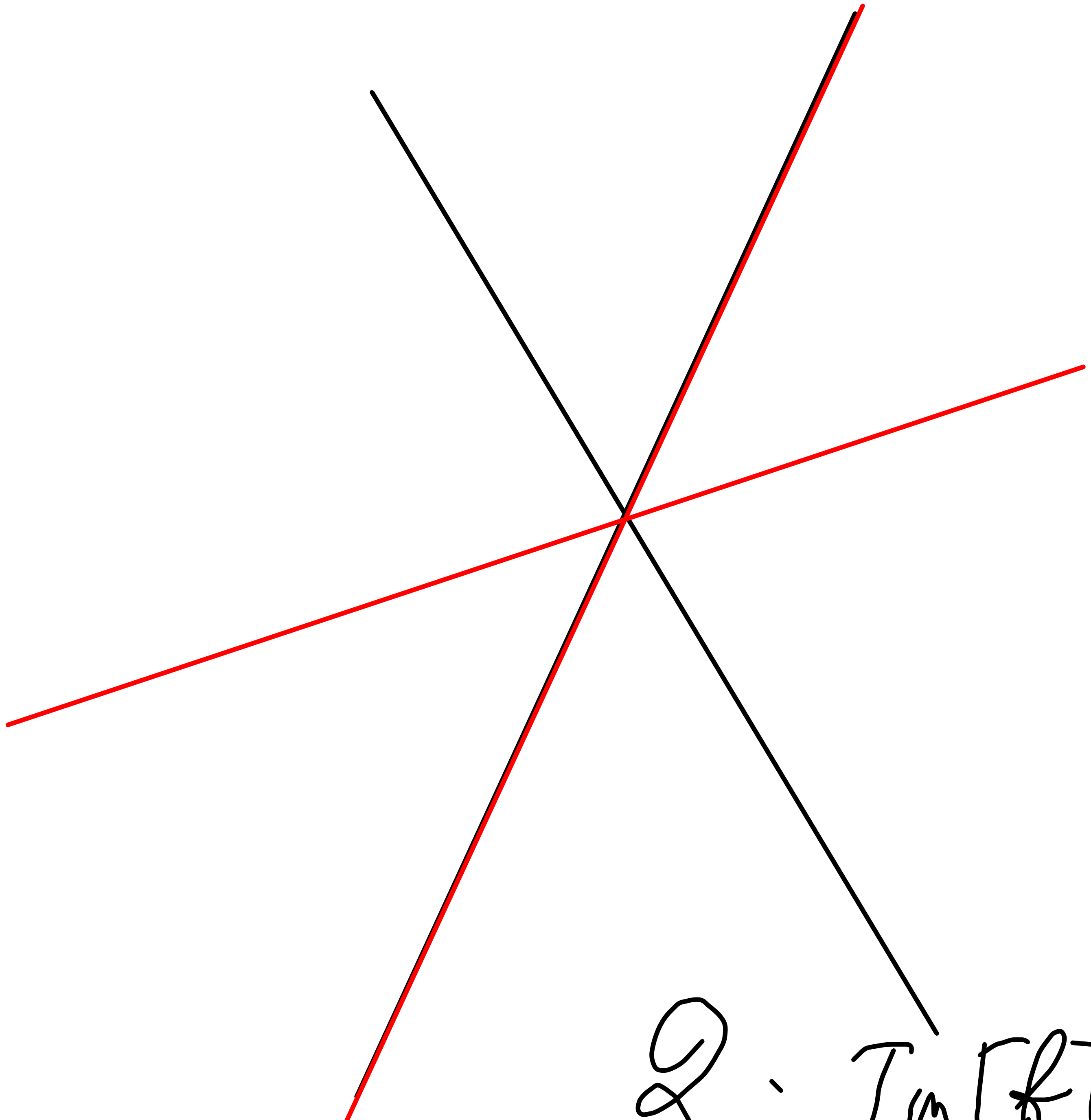
$\begin{cases} y = x^2 \\ x = 1 \end{cases}$ $\begin{cases} y = 1 \\ x = 1 \end{cases}$ $\begin{cases} X_2 X_0 = X_1^2 \\ X_0 = 0 \end{cases}$ $\begin{cases} X_1^2 = 0 \\ X_0 = 0 \end{cases}$ $\begin{cases} X_2 X_0 = X_1^2 \\ X_1 = X_0 \end{cases}$ $\begin{cases} X_2 X_0 = X_0^2 \\ X_1 = X_0 \end{cases}$

$Q: x^2 - y^2 = 0$

$(x+y)(x-y) = 0 \qquad y = 0$

$$\begin{cases} X_1^2 - X_2^2 = 0 \\ X_2 = 0 \end{cases}$$

$$\begin{cases} X_1^2 = 0 \\ X_2 = 0 \end{cases} \qquad (1,0,0)$$



$Q: \mathrm{Im}[f]$

Ho un'iperquadrica $[f]$ con immagine $\mathcal{Q}$, ho due punti $\overline{P} \neq \overline{Q}$
$\overline{P} \equiv (\overline{X}_0, \dots, \overline{X}_n)$, $\overline{Q} \equiv (\overline{Y}_0, \dots, \overline{Y}_n)$. Sia r la retta contenente
$\overline{P}$ e $\overline{Q}$. Cerco l'intersezione $r \cap \mathcal{Q}$
Il generico punto di r ha coordinate della forma

$$\lambda(\overline{X}) + \mu(\overline{Y}) \qquad \text{con } (\lambda, \mu) \neq (0,0)$$

L'equazione di $\mathcal{Q}$ è:

$$(Z_0 \cdots Z_n) \cdot A \cdot \begin{pmatrix} Z_0 \\ \vdots \\ Z_n \end{pmatrix} = 0$$

$$^t(Z) \cdot A \cdot (Z) = 0$$

$r \cap Q$ mi viene data dalle coppie $(\lambda, \mu) \neq (0,0)$ che soddisfano l'equazione data dalla composizione:

$r: (Z) = \lambda(\overline{X}) + \mu(\overline{Y})$

$Q: {}^t(Z) \cdot A \cdot (Z) = 0$

$r \cap Q: \begin{cases} \text{—} \\ \text{—} \end{cases}$

$$r \cap Q: \begin{cases} (Z) = \lambda(\overline{X}) + \mu(\overline{Y}) \\ {}^t\left(\lambda(\overline{X}) + \mu(\overline{Y})\right) \cdot A \cdot \left(\lambda(\overline{X}) + \mu(\overline{Y})\right) = 0 \end{cases}$$

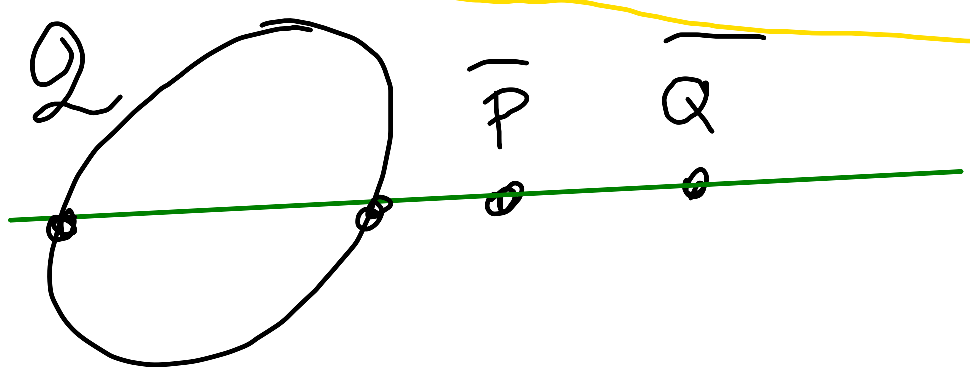
$${}^t\left(\lambda(\bar{X})+\mu(\bar{Y})\right)\cdot A\cdot\left(\lambda(\bar{X})+\mu(\bar{Y})\right)=0$$

$$\lambda\,{}^t(\bar{X})\cdot A\cdot\left(\lambda(\bar{X})+\mu(\bar{Y})\right)+\mu\,{}^t(\bar{Y})\cdot A\cdot\left(\lambda(\bar{X})+\mu(\bar{Y})\right)=0$$

$$\lambda\,{}^t(\bar{X})\cdot A\cdot\lambda\cdot(\bar{X})+\lambda\,{}^t(\bar{X})\cdot A\cdot\mu(\bar{Y})+\mu\,{}^t(\bar{Y})\cdot A\cdot\lambda(\bar{X})+\mu\,{}^t(\bar{Y})\cdot A\cdot\mu(\bar{Y})=0$$

$$=$$

$$\lambda^2\,{}^t(\bar{X})\cdot A\,(\bar{X})+2\lambda\mu\,{}^t(\bar{X})\cdot A\cdot(\bar{Y})+\mu^2\,{}^t(\bar{Y})\cdot A\cdot(\bar{Y})=0$$



$$k=\frac{\lambda}{\mu}$$

$$k^2\,{}^t(\bar{X})\cdot A(\bar{X})+2k\,{}^t(\bar{X})A(\bar{Y})+{}^t(\bar{Y})A(\bar{Y})=0$$

in tal caso i punti Q coniugati a P sono tutti e soli quelli le cui coordinate soddisfano

$$b_0 X_0 + b_1 X_1 + \dots + b_n X_n = 0$$

non tutti nulli

un iperpiano

$$ {}^t\left((X_0 \dots X_n) \cdot A \cdot \begin{pmatrix} \bar X_0 \\ \vdots \\ \bar X_n \end{pmatrix} \right) = (\bar X_0 \dots \bar X_n)\, {}^tA \cdot \begin{pmatrix} X_0 \\ \vdots \\ X_n \end{pmatrix} = $$

$$ = (\bar X_0 \dots \bar X_n) \cdot A \cdot \begin{pmatrix} X_0 \\ \vdots \\ X_n \end{pmatrix} $$