Classificare le quadriche

$$(3\lambda-1)x^2+(1-\lambda)y^2+2\lambda xz+2z-4\lambda=0$$

$$A=\begin{pmatrix}-4\lambda & 0 & 0 & 1\\ 0 & (3\lambda-1) & 0 & \lambda\\ 0 & 0 & (1-\lambda) & 0\\ 1 & \lambda & 0 & 0\end{pmatrix} \qquad |A|=(1-\lambda)(\lambda+1)(2\lambda-1)^2$$

$(1-\lambda)$ ———————— 0 - - - -

$(\lambda+1)$ - - - 0 ————————

$(2\lambda-1)^2$ ———— 0 ————

$-1 \qquad \frac{1}{2} \qquad 1$

$|A|$ $\quad - \quad 0 \quad + \quad 0 \quad + \quad 0 \quad -$

ell. deg $r=3$ $\qquad$ deg $r=3$ $\qquad$ deg ell $r=3$

$$\lambda=1\quad \begin{pmatrix}-4 & 0 & 0 & 1\\ 0 & 2 & 0 & 1\\ 0 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\end{pmatrix} \qquad \begin{vmatrix}-4 & 0 & 1\\ 0 & 2 & 1\\ 1 & 1 & 0\end{vmatrix}=\begin{vmatrix}-4 & 4 & 1\\ 0 & 2 & 1\\ 1 & 0 & 0\end{vmatrix}\neq 0$$

$|M_0^0|=0$ cilindro reale

$$\begin{pmatrix}-4 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}-4 & 0 & 1\\ 0 & 2 & 1\\ 1 & 1 & 0\end{pmatrix}$$

$$\lambda=\frac{1}{2}\quad \begin{pmatrix}-2 & 0 & 0 & 1\\ 0 & \frac{1}{2} & 0 & \frac{1}{2}\\ 0 & 0 & \frac{1}{2} & 0\\ 1 & \frac{1}{2} & 0 & 0\end{pmatrix} \qquad |M_0^0|\neq 0 \quad \text{cono reale}$$

$$\lambda=-1\quad \begin{pmatrix}4 & 0 & 0 & 1\\ 0 & -4 & 0 & -1\\ 0 & 0 & 2 & 0\\ 1 & -1 & 0 & 0\end{pmatrix} \qquad |M_0^0|\neq 0 \quad \text{cono reale}$$

$$A = \begin{pmatrix} a_0^0 & a_1^0 & \cdots & a_n^0 \\ a_0^1 & a_1^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots & \cdots \\ a_0^n & a_1^n & \cdots & a_n^n \end{pmatrix} \qquad M_\alpha^0$$

Cerco i poli degli iperp. principali: cerco un punto improprio $P_\infty = (0, l_1, \ldots, l_n)$ tale che il suo iperp. polare sia ortog. alla direzione da lui (P_∞) rappresentata

$$\tau(P_\infty): \begin{pmatrix} X_0 & X_1 & \cdots & X_n \end{pmatrix} \begin{pmatrix} a_0^0 & a_1^0 & \cdots & a_n^0 \\ a_0^1 & a_1^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots & \cdots \\ a_0^n & a_1^n & \cdots & a_n^n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$$

$$= \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{pmatrix}$$

$c_0X_0 + c_1X_1 + \cdots + c_nX_n = 0 \perp P_\infty \iff (c_1, \ldots, c_n) \sim (l_1, \ldots, l_n)$

$\iff \exists \lambda \neq 0$ tale che

$$\begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = M_\alpha^0 \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

a) Verificare che $\Gamma: 3x^2-10xy+3y^2+8x-8y+6=0$ è un'iperbole.

b) Trovarne asintoti, assi e centro.

a) $A=\begin{pmatrix} 6 & 4 & -4 \\ 4 & \boxed{3} & \boxed{-5} \\ -4 & \boxed{-5} & \boxed{3} \end{pmatrix}$ M_0^0 $\quad |A| = \begin{vmatrix} 6 & 4 & 0 \\ 4 & 3 & -2 \\ -4 & -5 & -2 \end{vmatrix} = \begin{vmatrix} 6 & 4 & 0 \\ 8 & 8 & 0 \\ -4 & -5 & -2 \end{vmatrix} = -2 \begin{vmatrix} 6 & 4 \\ 8 & 8 \end{vmatrix} \neq 0$ non deg

$|M_0^0| = 9-25 < 0$ iperbole

b) $\begin{cases} 3X_1^2-10X_1X_2+3X_2^2 + \cancel{8X_1X_0} - \cancel{8X_2X_0} + \cancel{6X_0^2} = 0 \\ X_0=0 \end{cases}$ $\quad 3X_1^2-10X_1X_2+3X_2^2=0$

$X_1 = X_2 \dfrac{5 \pm \sqrt{25-9}}{3} = X_2 \dfrac{5\pm 4}{3} = \begin{cases} 3X_2 \\ \frac{1}{3}X_2 \end{cases}$ $\quad X_0=0$

Intersezione $\Gamma \cap r_\infty$: $P_\infty \equiv (0,3,1)$ $\quad Q_\infty \equiv (0,1,3)$

Polari $\tau(P_\infty)$: $(0\ 3\ 1)\begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$ $\quad 8+4x-12y=0$ $\quad x-3y+2=0$

$\tau(Q_\infty)$: $(0\ 1\ 3)$ $\quad -8-12x+4y=0$ $\quad 3x-y+2=0$

centro: intersez. degli asintoti

Assi $M_0^0 = \begin{pmatrix} 3 & -5 \\ -5 & 3 \end{pmatrix}$ $\quad \begin{vmatrix} (\lambda-3) & 5 \\ 5 & (\lambda-3) \end{vmatrix} = (\lambda-3)^2-5^2 = (\lambda-3+5)(\lambda-3-5) = (\lambda+2)(\lambda-8)$

autovalori $-2, 8$

$\lambda=-2$ $\quad U_{-2}\begin{pmatrix} \cancel{5} & \cancel{5} \\ 5 & -5 \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \end{pmatrix}$ $\quad 5l-5m=0$ $\quad R_\infty \equiv (0,1,1)$

$\lambda=8$ $\quad U_8 \begin{pmatrix} 5 & 5 \\ \cancel{5} & \cancel{5} \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \end{pmatrix}$ $\quad 5l+5m=0$ $\quad S_\infty \equiv (0,1,-1)$

$$A = \begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix} \quad R_\infty \equiv (0,1,1)$$
$$S_\infty \equiv (0,1,-1)$$

$$a_1 = \tau(R_\infty): (0\ 1\ 1)\begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0 \qquad -2x - 2y = 0 \qquad x + y = 0$$

$$a_2 = \tau(S_\infty): (0\ 1\ -1) \qquad 8 + 8x - 8y = 0 \qquad x - y + 1 = 0$$

centro: intersezione degli assi

(R_1) trovo gli assi senza usare il teor. sugli autovalori

Generico punto improprio $T_\infty \equiv (0, l, m)$

$$\tau(T_\infty): (0\ l\ m)\begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0 \qquad (4l - 4m) + \underset{a_1}{(3l-5m)}x + \underset{a_2}{(-5l+3m)}y = 0$$

$$\perp \iff \begin{pmatrix} 3l-5m \\ -5l+3m \end{pmatrix} \sim \begin{pmatrix} l \\ m \end{pmatrix} \iff m(3l-5m) = l(-5l+3m) \iff -5l^2 + \cancel{3lm} \iff$$

$$\iff \cancel{3lm} - 5m^2 = -5l^2 + \cancel{3lm} \iff$$

$$\iff l^2 = m^2 \iff l = \pm m$$

C'è un altro modo per trovare il centro: interseco le polari di due punti impropri qualunque

Per pigrizia scelgo $B_\infty \equiv (0,1,0)$

$C_\infty \equiv (0,0,1)$

$$A = \begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix}$$

$\pi(B_\infty): (0\ 1\ 0)$

$\pi(C_\infty): (0\ 0\ 1)$

$$\begin{pmatrix} 6 & 4 & -4 \\ 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = 0 \quad \begin{cases} 4X_0 + 3X_1 - 5X_2 = 0 \\ -4X_0 - 5X_1 + 3X_2 = 0 \end{cases}$$

$$\begin{pmatrix} 4 & 3 & -5 \\ -4 & -5 & 3 \end{pmatrix}$$

Centro:

$$(\overline{X}_0, \overline{X}_1, \overline{X}_2) \sim \left(\overset{A_0^0}{\begin{vmatrix} 3 & -5 \\ -5 & 3 \end{vmatrix}}, -\overset{A_1^0}{\begin{vmatrix} 4 & -5 \\ -4 & 3 \end{vmatrix}}, \overset{A_2^0}{\begin{vmatrix} 4 & 3 \\ -4 & -5 \end{vmatrix}} \right) =$$

$$= (-16, 8, -8) \sim (2, -1, 1)$$

$$\text{Centro} \equiv \left(-\frac{1}{2}, \frac{1}{2}\right)$$

In $\mathcal{E}^2$

$x^2+5y^2+1=0$ ha 2 assi

$2x^2+2y^2+1=0$ ne ha ∞

è di rotazione

In $\mathcal{E}^3$

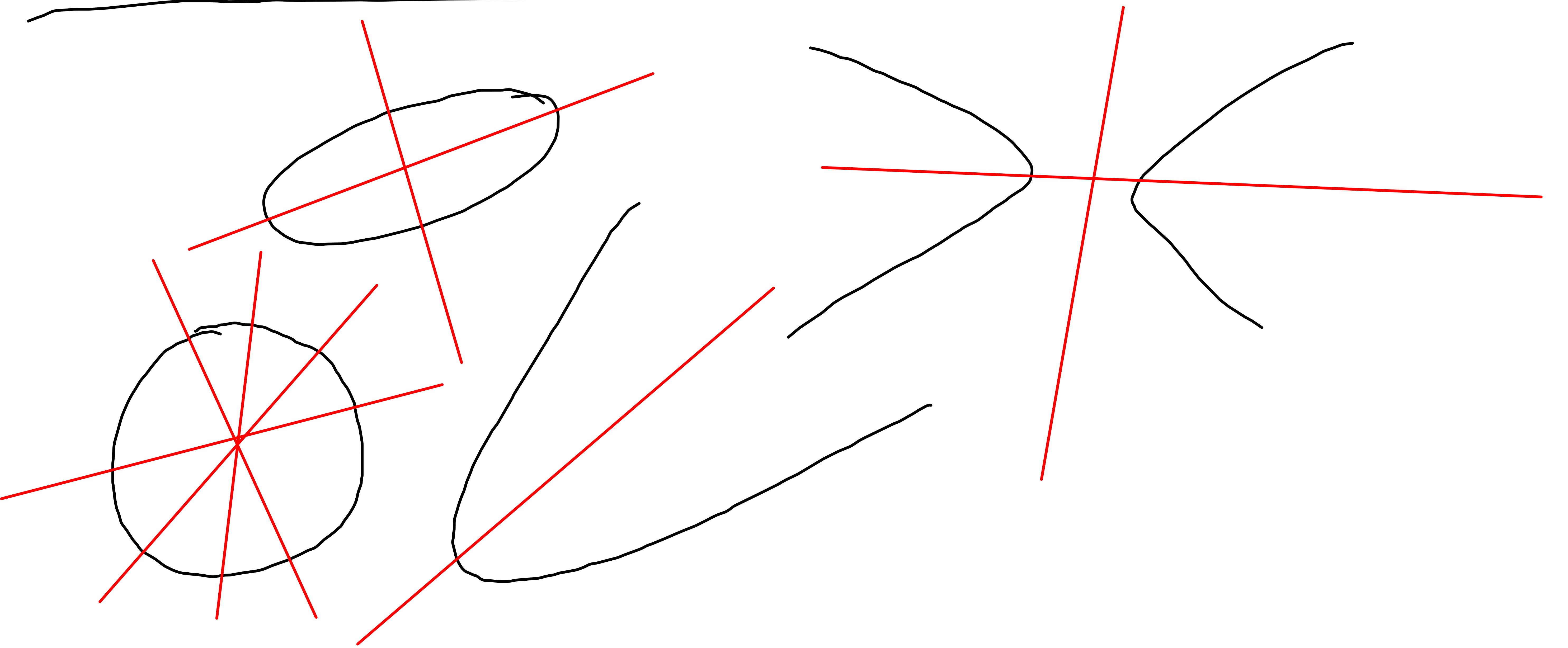
$x^2+5y^2+7z^2+1=0$ ha 3 piani principali

$x^2+5y^2+5z^2+1=0$ ha 1 piano principale per conto suo e ∞ piani principali per una certa retta

$5x^2+5y^2+5z^2+1=$ ha tutti i piani diametrali principali

Trovare i piani principali e l'eventuale spazio di rotazione della quadrica

$$3x^2+2y^2+2z^2+2yz+6y+6z-1=0$$

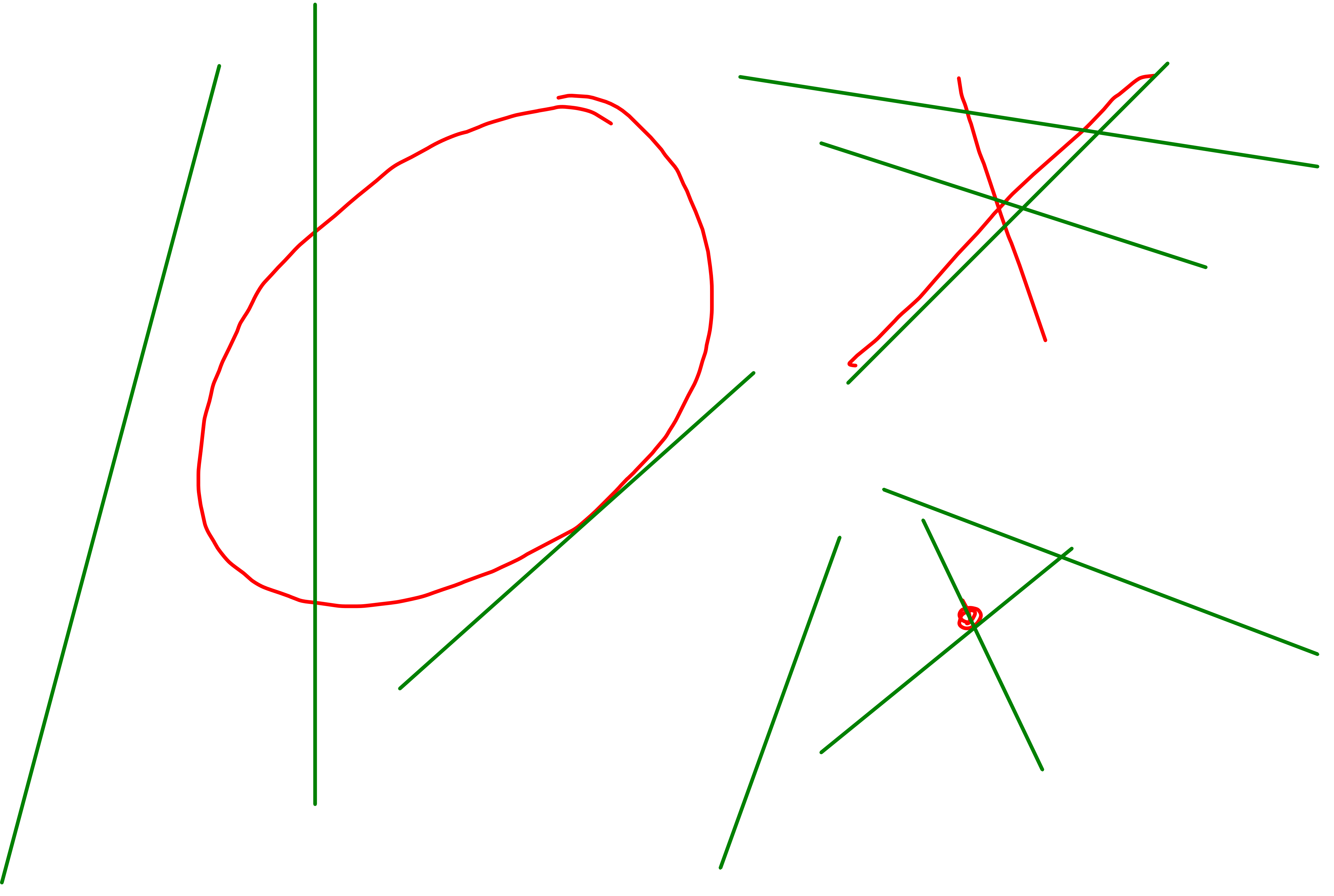


$$\begin{pmatrix} \cdot & - & \backslash & \backslash \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \to \begin{pmatrix} a & & 0 \\ & b & \\ 0 & & c \end{pmatrix}$$

par. ip. $\begin{pmatrix} 1 & & 0 \\ & -2 & \\ 0 & & 0 \end{pmatrix}$ par. ell. $\begin{pmatrix} 1 & & 0 \\ & 2 & \\ 0 & & 0 \end{pmatrix}$ $\begin{pmatrix} 3 & & 0 \\ & 3 & \\ 0 & & 0 \end{pmatrix}$

ip. ip. $-1\begin{pmatrix} 1 & & \\ & 2 & \\ & & -3 \end{pmatrix}$ $-1\begin{pmatrix} 2 & & \\ & 2 & \\ & & -3 \end{pmatrix}$ ip. ell. $1\begin{pmatrix} 1 & & \\ & 2 & \\ & & -3 \end{pmatrix}$ $1\begin{pmatrix} 2 & & \\ & 2 & \\ & & -3 \end{pmatrix}$

ell. $\pm 1\begin{pmatrix} 1 & & \\ & 2 & \\ & & 3 \end{pmatrix}$ $\pm 1\begin{pmatrix} 2 & & \\ & 2 & \\ & & 3 \end{pmatrix}$ $\pm 1\begin{pmatrix} 3 & & \\ & 3 & \\ & & 3 \end{pmatrix}$



$$A = \begin{pmatrix} -4\lambda & 0 & 0 & 1 \\ 0 & (3\lambda+1) & 0 & \lambda \\ 0 & 0 & (1-\lambda) & 0 \\ 1 & \lambda & 0 & 0 \end{pmatrix} \qquad A_0^0 = |M_0^0| = \lambda \begin{vmatrix} 0 & (1-\lambda) \\ \lambda & 0 \end{vmatrix} = \lambda^2(\lambda-1) = 0 \iff \begin{cases} \lambda = 0 \\ \lambda = 1 \end{cases}$$

* mi dice che M_0^0 è sempre indefinita

	$\lvert A\rvert$	$r(A)$	M_0^0	Q_∞	quadriche
$\lambda<-1$	$-$	4	$\neq 0$ *	ndr	iperboloidi ellittici
$\lambda=-1$	0	3	$\neq 0$ *	ndr	cono r
$-1<\lambda<0$	$+$	4	$\neq 0$ *	ndr	iperboloidi iperbolici
$\lambda=0$	$+$	4	$=0$	deg	paraboloide iperbolico
$0<\lambda<\frac{1}{2}$	$+$	4	$\neq 0$ *	ndr	iperboloidi iperbolici
$\lambda=\frac{1}{2}$	0	3	$\neq 0$ *	ndr	cono r
$\frac{1}{2}<\lambda<1$	$+$	4	$\neq 0$ *	ndr	iperboloidi iperbolici
$\lambda=1$	0	3	$=0$	deg	cilindro r
$\lambda>1$	$-$	4	$\neq 0$ *	ndr	iperboloidi ellittici