$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)f(x_0,y_0)$$ sta per

$$(x-x_0)\frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}+(y-y_0)\frac{\partial f}{\partial y}(x_0,y_0)$$

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)$$ sta per

$$(x-x_0)^2\frac{\partial^2 f}{\partial x^2}\Big|_{(x_0,y_0)}+2(x-x_0)(y-y_0)\frac{\partial^2 f}{\partial x\,\partial y}\Big|_{(x_0,y_0)}+(y-y_0)^2\frac{\partial^2 f}{\partial y^2}\Big|_{(x_0,y_0)}$$

ecc.

$C: f(x,y)=0 \qquad P\equiv(x_0,y_0)\in C \qquad f(x_0,y_0)=0$

x_0 di molt. $= s$ $\iff$ $=0 \ \forall k$

$$\begin{cases} f(x,y) = \left((x-x_0)\frac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\frac{\partial}{\partial y}\right) f(x_0,y_0) + \\ y-y_0=k(x-x_0) \end{cases}$$

$$+\frac{1}{2}\left((x-x_0)\frac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) +$$

$$\vdots$$

$$+\frac{1}{(s-1)!}\left((x-x_0)\frac{\partial}{\partial x} + \overset{k(\ )}{(y-y_0)}\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0) +$$

$$+\frac{1}{s!}\left((x-x_0)\frac{\partial}{\partial x} + \overset{k(\ )}{(y-y_0)}\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0) +$$

$$+\frac{1}{(s+1)!}\left((x-x_0)\frac{\partial}{\partial x} + \overset{k(\ )}{(y-y_0)}\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0) +$$

$$+ \cdots$$

$$\begin{cases} y-y_0=k(x-x_0) \\ 0 = \boxed{(x-x_0)\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right) f(x_0,y_0) + \frac{(x-x_0)^2}{2}\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) + \cdots + \frac{(x-x_0)^{s-1}}{(s-1)!}\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0)} + \\ + \frac{(x-x_0)^{s}}{s!}\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0) + \\ + \frac{(x-x_0)^{s+1}}{(s+1)!}\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0) + \\ \cdots \end{cases}$$

Per quali k x_0 e' di molt. $\geq s+1$?

quando $=0$

Es 4 20/7/'10

$e : \begin{cases} x = \alpha^2 \\ y = \alpha^3 - \alpha \end{cases}$

a) eq. cart. di e

b) p multipli e tang.

a)

$$\begin{array}{l|l} \alpha^3 - \alpha - y & \alpha^2 - x \\ \hline -\alpha^3 + \alpha x & \alpha \\ \hline // \quad \alpha(x-1) - y & \end{array}$$

$$\begin{array}{l|l} \alpha^2 - x & \alpha(x-1) - y \\ \hline -\alpha^2 + \frac{\alpha y}{x-1} & \frac{\alpha}{x-1} + \frac{y}{(x-1)^2} \\ \hline // \quad \frac{\alpha y - x(x-1)}{x-1} & \\ -\frac{\alpha y}{x-1} + \frac{y^2}{(x-1)^2} & \\ \hline // \quad \frac{y^2 - x(x-1)^2}{(x-1)^2} & \end{array}$$

$$\boxed{e : y^2 - x^3 + 2x^2 - x = 0}$$

b) $C : \underbrace{y^2 - x^3 + 2x^2 - x}_{f(x,y) =} = 0$

$f_x = -3x^2 + 4x - 1 = 0 \iff x = 1 \lor x = \frac{1}{3}$

$f_y = 2y = 0 \iff y = 0$

$$\begin{cases} f = 0 \\ f_x = 0 \\ f_y = 0 \end{cases}$$

$M = \boxed{(1,0)}$ $\begin{cases} f = 0 - 1 + 2 - 1 = 0 \\ f_x = 0 \\ f_y = 0 \end{cases}$

$\xcancel{\left(\frac{1}{3}, 0\right)}$ $\begin{cases} f = 0 - \frac{1}{27} + \frac{2}{9} - \frac{1}{3} = \frac{-1+6-9}{27} \neq 0 \\ f_x = 0 \\ f_y = 0 \end{cases}$

	in M
$f_{xx} = -6x + 4$	-2
$f_{xy} = 0$	0
$f_{yy} = 2$	2

Complesso delle tang. in M

$$\left((x-1)\frac{\partial}{\partial x} + (y-0)\frac{\partial}{\partial y}\right)^2 f(1,0) = 0$$

$$-2(x-1)^2 + 2\cdot 0(x-1)(y-0) + 2(y-0)^2 = 0$$

$$-2(x-1)^2 + 2y^2 = 0$$

Le singole tang.: $y = k(x-1)$

con $\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(1,0) = 0$

$-2 + 2\cdot 0\cdot k + 2\cdot k^2 = 0$

$2k^2 - 2 = 0$ $k^2 = 1$

$k = \pm 1$

$t_1 : y = x - 1$ $t_2 : y = -x + 1$

Verificare (3,0) multiplo e tang. di P: $f(x,y) = \boxed{4y^3+4x^2y-12xy+x^2-6x+9} = 0$

$f_x = 8xy - 12y + 2x - 6$

$f_y = 12y^2 + 4x^2 - 12x$

$(3,0)$ $0+0+0+9-18+9=0$ ✓

$0-0+6-6=0$ ✓

$0+36-36=0$ ✓ multiplo

$(3,0)$

$f_{xx} = 8y+2$ $\quad 2$

$f_{xy} = 8x-12$ $\quad 12$

$f_{yy} = 24y$ $\quad 0$

Complesso delle tang.

$$\left((x-3)^2 \cdot 2 + 2 \cdot 12(x-3)y + 0 \cdot y^2\right) = 0$$

Trova le 2 tang.

$$\left(2 + 2 \cdot 12k + 0k^2\right) = 0$$

$$24k + 2 = 0 \qquad k = -\frac{1}{12}$$

$y - y_0 = k(x - x_0)$

$k = \frac{\beta}{\alpha}$

$\alpha(y - y_0) = \beta(x - x_0)$

$\alpha y = \beta(x-3)$

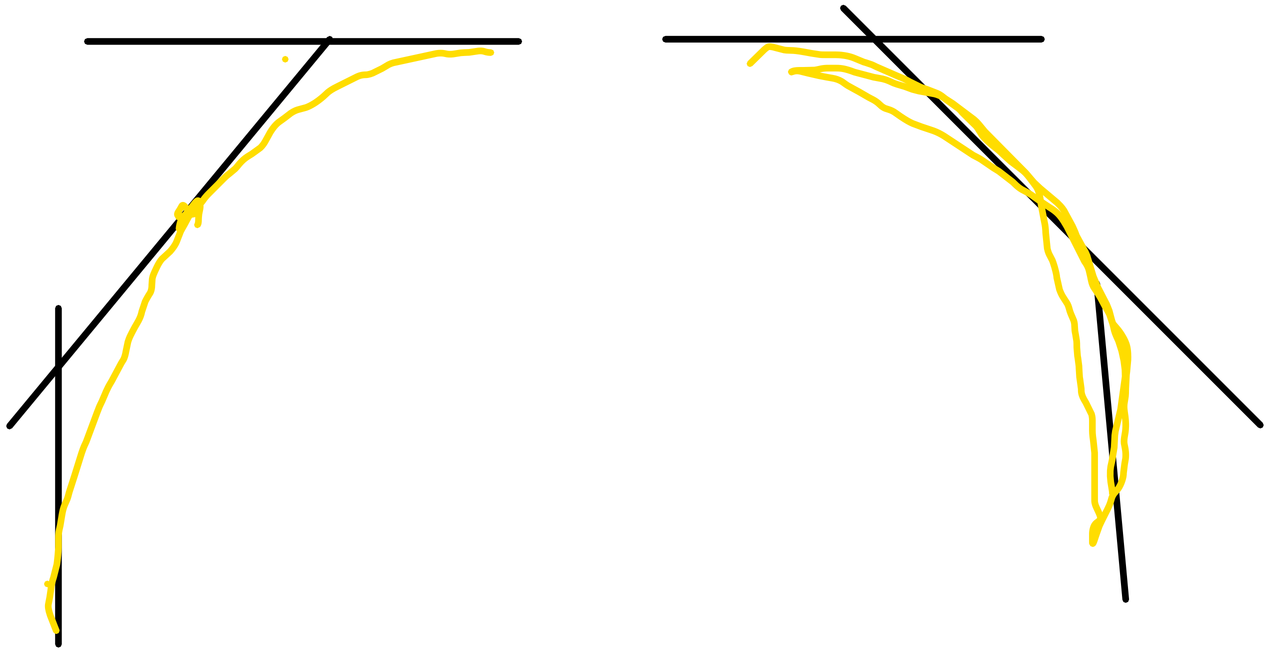
$$\left(2\alpha^2 + 2 \cdot 12\alpha\beta + 0 \cdot \beta^2\right) = 0$$

$$2\alpha^2 + 24\alpha\beta = 0 \qquad 2\alpha(\alpha + 12\beta) = 0$$

$\alpha = 0 \rightarrow x - 3 = 0$

$\beta = -\frac{1}{12}\alpha$

$y = -\frac{1}{2}(x-3)$



20/3/'09 es 4   Inviluppo di $\boxed{3ax^2 - 3a^2x - y + a^3} = 0$

$f(x, y; a) =$

$f_a = 3x^2 - 6ax + 3a^2$

$a^3 - 3xa^2 + 3x^2a - y$

$x^3 - y$

$$\begin{array}{|l} 3a^2 - 6xa + 3x^2 \\ \hline a - x \\ \overline{\quad 3 \quad} \end{array}$$

$$\boxed{y - x^3 = 0}$$

$x^2+(y-a)^2=1$

$$\boxed{x^2+y^2-2ay+a^2-1=0}$$

$f(x,y,a)=$

$f_a=-2y+2a$

$\begin{cases} f=0 \\ f_a=0 \end{cases}$ $\begin{cases} f=0 \\ a=y \end{cases}$

$x^2+\cancel{y^2}-\cancel{2y^2}+\cancel{y^2}-1=0$ $(x+1)(x-1)=0$

$x^2+(y-a^2)=1$

$x^2+y^2-2a^2y+a^4-1=0$

$f=$

$f_a=-4ay+4a^3=4a(a^2-y)$

$\begin{cases} f=0 \\ f_a=0 \end{cases}$ $\begin{cases} f=0 \\ 4a(a^2-y)=0 \end{cases}$

$a=0$ $x^2+y^2-1=0$

$y=a^2$

$x^2+\cancel{y^2}-\cancel{2y^2}+\cancel{y^2}-1=0$