Es 6 $\Sigma : \begin{cases} x = u^2 - uv \\ y = v \\ z = u - v^2 \end{cases}$

a) eq. cart.
b) tang. as.
c) luogo punti in cui la norm. è $\parallel \pi : 2x + z = 0$, verif. piana

a) $\begin{cases} x = u^2 - uy \\ z = u - y^2 \end{cases}$ $x = (z+y^2)^2 - (z+y^2)y$

$u = z + y^2$ $x = \underbrace{z^2 + y^4 + 2y^2z - yz - y^3 - x}_{F(x,y,z)}$

$O = (0,0,0)$ è punto semplice

Piano tang. in O: $x = 0$

Tangenti in O: $\begin{cases} x = 0 \\ z = ky \end{cases}$ $\begin{cases} x = 0 \\ y = \alpha \\ z = k\alpha \end{cases}$ $O \leftrightarrow \alpha = 0$

$\Phi(\alpha) = k^2\alpha^2 + \alpha^4 + 2k\alpha^3 - k\alpha^2 - \alpha^3$

$\Phi'(\alpha) = 2k^2\alpha + 4\alpha^3 + 6k\alpha^2 - 2k\alpha - 3\alpha^2$

$\Phi''(\alpha) = 2k^2 + 12\alpha^2 + 12k\alpha - 2k - 6\alpha$

$\begin{cases} \Phi(0) = 0 \\ \Phi'(0) = 0 \\ \Phi''(0) = 0 \end{cases}$ $\forall k$ $\Phi'(0) = 0$

$\Phi''(0) = 0 \iff 2k^2 - 2k = 0$ $2k(k-1) = 0$ $\begin{cases} k = 0 \\ k = 1 \end{cases}$

$\begin{cases} x = 0 \\ z = 0 \end{cases}$ $\begin{cases} x = 0 \\ z = y \end{cases}$

c) $\Sigma: \quad x = \underbrace{z^2 + y^4 + 2y^2 z - yz - y^3}_{G(y,z)=}$

$$\begin{cases} x = \beta^2 + \alpha^4 + 2\alpha^2\beta - \alpha\beta - \alpha^3 \\ y = \alpha \\ z = \beta \end{cases}$$

$P_{\alpha\beta} = (\beta^2 + \alpha^4 + 2\alpha^2\beta - \alpha\beta - \alpha^3, \ \alpha, \ \beta)$

$P: \begin{cases} x = \frac{\alpha^2}{4} + 1 \\ y = \alpha \\ z = 1 - \alpha^2 + \alpha/2 \end{cases}$

$G_y = 4y^3 + 4yz - z - 3y^2$ in $P_{\alpha\beta}$: $4\alpha^3 + 4\alpha\beta - \beta - 3\alpha^2$

$G_z = 2z + 2y^2 - y$ — $2\beta + 2\alpha^2 - \alpha$

Piano tang. in $P_{\alpha\beta}$: $\Pi_{\alpha\beta}: -(x - (\beta^2 + \alpha^4 + 2\alpha^2\beta - \alpha\beta - \alpha^3)) + (4\alpha^3 + 4\alpha\beta - \beta - 3\alpha^2)(y - \alpha) + (2\beta + 2\alpha^2 - \alpha)(z - \beta) = 0$

norm. in $P_{\alpha\beta}$

$$n_{\alpha\beta}: \frac{x - (\beta^2 + \alpha^4 + 2\alpha^2\beta - \alpha\beta - \alpha^3)}{-1} = \frac{y - \alpha}{4\alpha^3 + 4\alpha\beta - \beta - 3\alpha^2} = \frac{z - \beta}{2\beta + 2\alpha^2 - \alpha}$$

$\parallel 2x + z = 0 \iff 2 \cdot (-1) + 0 \cdot (4\alpha^3 + 4\alpha\beta - \beta - 3\alpha^2) + 1 \, (2\beta + 2\alpha^2 - \alpha) = 0 \quad 2\beta + 2\alpha^2 - \alpha - 2 = 0$

$$\frac{x - \frac{\alpha^2}{4} - 1}{-1} = \frac{y - \alpha}{\frac{7\alpha}{2} - 1} = \frac{z - 1 + \alpha^2 - \alpha/2}{2}$$

$$ax+by+cz+d=0$$

$F=$

$$\Phi(\alpha) = a\left(\frac{\alpha^2}{4}+1\right) + b\alpha + c\left(1-\alpha^2+\alpha/2\right) + d =$$

$$= \left(\frac{a}{4}-c\right)\alpha^2 + \left(b+\frac{c}{2}\right)\alpha + a+c+d \qquad = 0 \ \forall \alpha$$

$$\Longleftrightarrow \begin{cases} \frac{a}{4} - c = 0 \\ b + \frac{c}{2} = 0 \\ a+c+d = 0 \end{cases} \quad \begin{cases} c = \frac{a}{4} \\ b = -\frac{a}{8} \\ a + \frac{a}{4} + d = 0 \end{cases} \quad \begin{cases} = \\ = \\ d = -\frac{5a}{4} \end{cases}$$

$$a = 8 \quad b = -1 \quad c = 2 \quad d = -10$$

$$8x - y + 2z - 10 = 0$$

Es 7 $\quad \gamma: x^2+y^2+2\lambda xy+4\lambda^2x+4=0$ $\qquad A_\lambda=\begin{pmatrix}4 & 2\lambda^2 & 0\\ 2\lambda^2 & 1 & \lambda\\ 0 & \lambda & 1\end{pmatrix}$

λ luogo poli asse x

$O\equiv(1,0,0)\quad X_\infty\equiv(0,1,0)$

polari:

$$\begin{matrix}(1\ 0\ 0)\\(0\ 1\ 0)\end{matrix}\cdot A\cdot\begin{pmatrix}1\\x\\y\end{pmatrix}=0 \qquad \begin{cases}4+2\lambda^2x=0\\ 2\lambda^2+x+\lambda y=0\end{cases}$$

$$\begin{cases}\lambda^2x+2=0\\ \lambda^3y+2\lambda^4-2=0\end{cases}$$

$$\begin{cases}x=-\dfrac{2}{\lambda^2}\\ y=\dfrac{-2\lambda^2+\frac{2}{\lambda^2}}{\lambda}=-2\lambda+\dfrac{2}{\lambda^3}\end{cases}$$

elimino λ .....