Es 19a Classificare

$$(\lambda-1)x^2+(\lambda-1)y^2+(\lambda+1)z^2+4(\lambda-1)x-2(\lambda-1)y+4\lambda-5=0$$

$$A=\begin{pmatrix} (4\lambda-5) & 2(\lambda-1) & (1-\lambda) & 0 \\ 2(\lambda-1) & \boxed{(\lambda-1)} & 0 & 0 \\ (1-\lambda) & 0 & (\lambda-1) & 0 \\ 0 & 0 & 0 & (\lambda+1) \end{pmatrix}$$

M_1, M_2, M_3, $M_0^0 =$

		-1		$+1$	
$(\lambda-1)^2$	+	+	+	0	+
$(\lambda+1)$	−	0	+	+	+
$\lvert M_0^0\rvert$	−	0	+	0	+

$A_0^0 =$

$$\lvert M_0^0\rvert = (\lambda-1)^2(\lambda+1) = 0 \iff \lambda=\pm 1$$

$$\lvert A\rvert = -\lambda^4+\lambda^3+\lambda^2-\lambda = -\lambda(\lambda-1)^2(\lambda+1)$$

		-1		0		$+1$	
$-\lambda$	+	+	+	0	−	−	−
$(\lambda-1)^2$	+	+	+	+	+	0	+
$(\lambda+1)$	−	0	+	+	+	+	+
$\lvert A\rvert$	−	0	+	0	−	0	−

$\lvert M_1\rvert = \lambda-1$

$\lvert M_2\rvert = (\lambda-1)^2$

$\lvert M_3\rvert = (\lambda-1)^2(\lambda+1)$

		-1		$+1$	
$\lvert M_1\rvert$	−	−	−	0	+
$\lvert M_2\rvert$	+	+	+	0	+
$\lvert M_3\rvert$	−	0	+	0	+
	def. n. $\mp$		ind. $\mp$		$\pm$ def. p.

$$\lambda=-1\quad \begin{pmatrix} -9 & -4 & 2 & 0 \\ -4 & -2 & 0 & 0 \\ 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$|M_0^\circ|=0 \longrightarrow$ cilindro

$|M_1| = \det(-9) = -9 < 0$

$|M_2| = \det\begin{pmatrix} -9 & -4 \\ -4 & -2 \end{pmatrix} = 2 > 0$

$$\begin{vmatrix} -9 & -4 & 2 \\ -4 & -2 & 0 \\ 2 & 0 & -2 \end{vmatrix} = \begin{vmatrix} -7 & -4 & 0 \\ -4 & -2 & 0 \\ 2 & 0 & -2 \end{vmatrix} = -2 \begin{vmatrix} -7 & -4 \\ -4 & -2 \end{vmatrix} \neq 0$$

$+4 > 0$

rango $= 3$ $\quad |M_3| > 0$

$-$
$+$
$+$

intersez. con $X_3 = 0$ non vuota

$$\lambda=0\quad \begin{pmatrix} -5 & -2 & 1 & 0 \\ -2 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

rango $= 3$

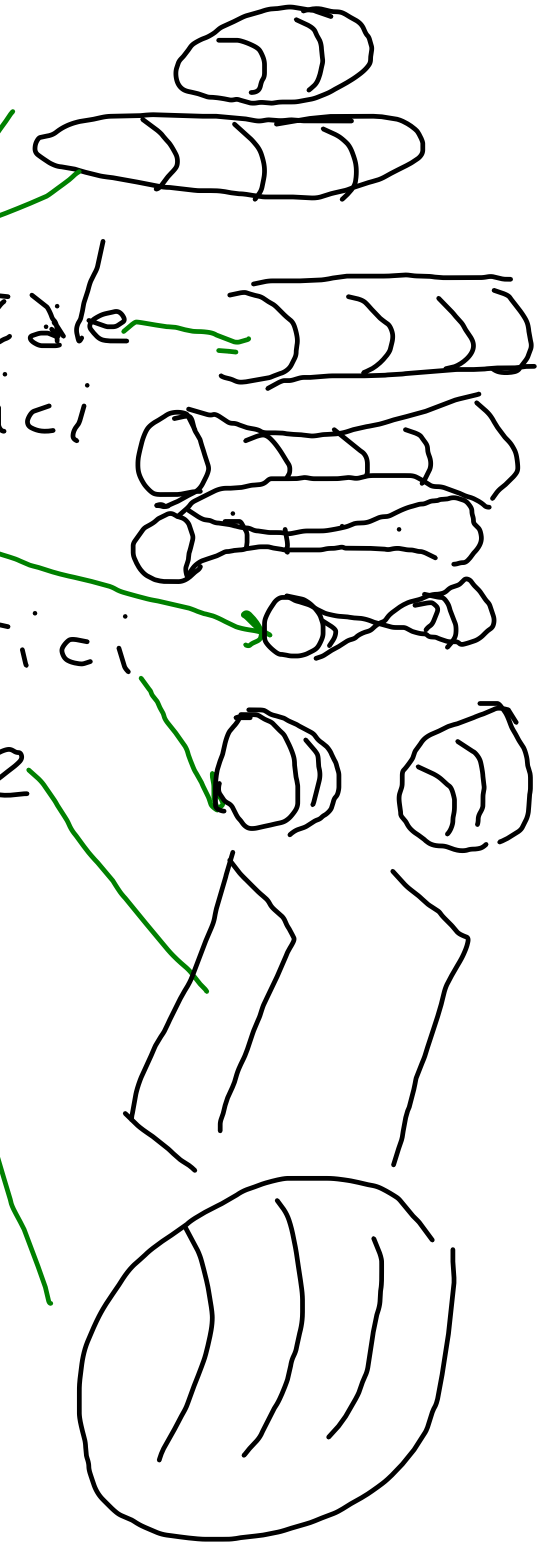
$|M_0^\circ| \neq 0 \longrightarrow$ cono reale

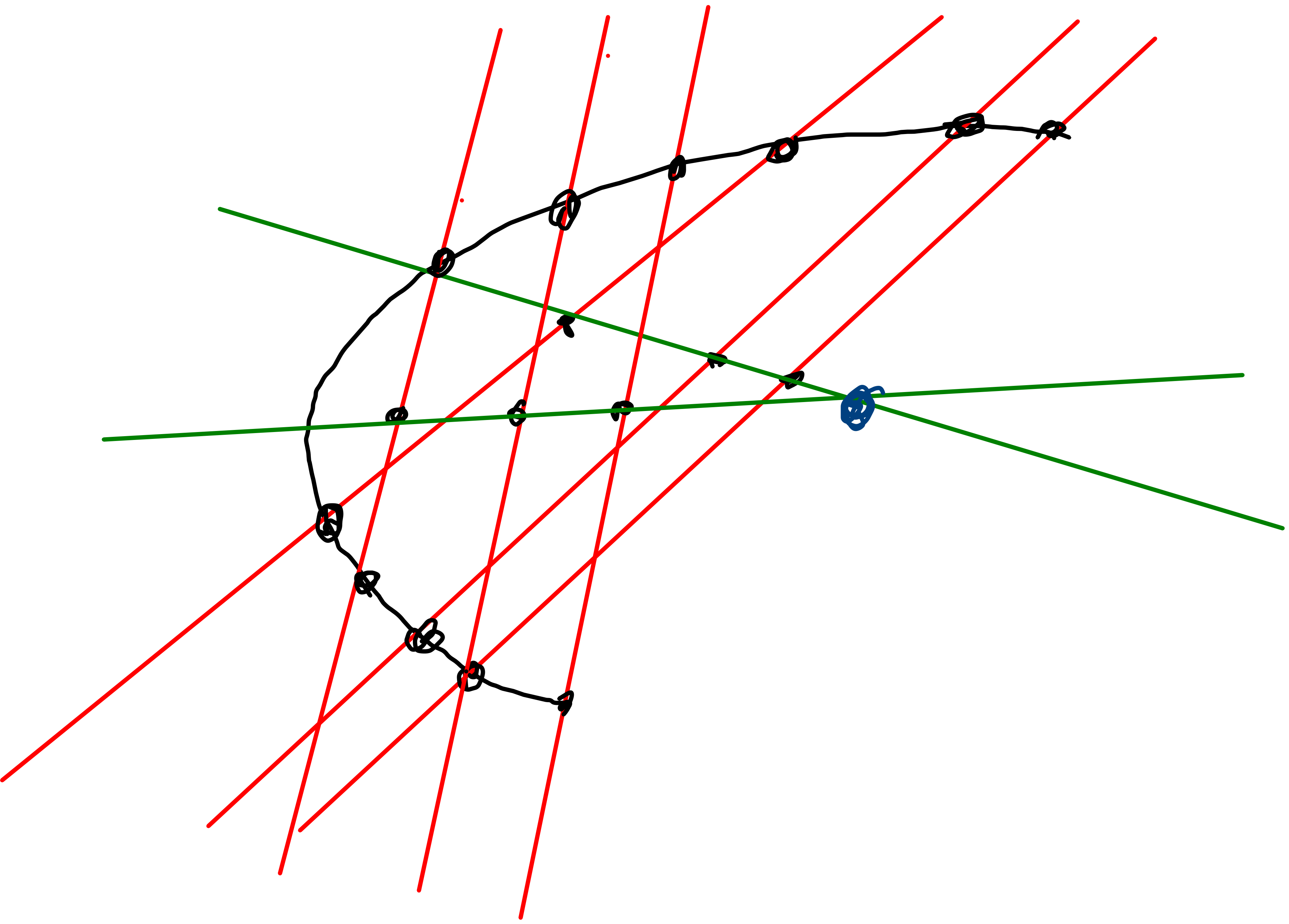
indef.

$$\lambda=+1\quad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

rango $= 2$

λ	$\lvert A \rvert$	rango A	Q_∞	quadriche
$\lambda < -1$	$-$	4	n.d.imm	ellissoidi reali
$\lambda = -1$	0	3	deg	cil. ~~immag~~ reale
$-1 < \lambda < 0$	$+$	4	n.d.re	iperboloidi iperbolici
$\lambda = 0$	0	3	n.d.re	cono reale
$0 < \lambda < 1$	$-$	4	n.d.re	iperboloidi ellittici
$\lambda = 1$	0	2	deg.	deg. di rango 2
$\lambda > 1$	$-$	4	n.d.imm.	ellissoidi reali





TEOR - [f] iperq. non spec. A discriminante.
$P_\infty = (0, l_1, \dots, l_n)$ è polo di un iperp. principale $\iff (l_1, \dots, l_n)$ è autovettore, relativo a un autovalore non nullo, di M_0^0

DIM - Sia $P_\infty = (0, l_1, \dots, l_n)$ un generico punto improprio
L'iperpiano diametrale suo polare è:

$$(0\ l_1 \dots l_n) \cdot A \cdot \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix} = 0 \quad \text{oppure} \quad (X_0\, X_1 \dots X_n) \cdot \begin{pmatrix} a_0^0 & a_1^0 & \dots & a_n^0 \\ a_0^1 & a_1^1 & \dots & a_n^1 \\ \vdots & & & \\ a_0^n & a_1^n & \dots & a_n^n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$$

$$(X_0\ X_1 \dots X_n) \cdot \begin{pmatrix} a_0^0 0 + a_1^0 l_1 + \dots + a_n^0 l_n \\ a_0^1 0 + a_1^1 l_1 + \dots + a_n^1 l_n \\ \dots \\ a_0^n 0 + a_1^n l_1 + \dots + a_n^n l_n \end{pmatrix} \begin{matrix} b \\ a_1 \\ \\ a_n \end{matrix}$$

$(1 \quad x_1 \dots x_n)$

$b + a_1 x_1 + \dots + a_n x_n$

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = M_0^0 \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

Condizione di ortogonalità retta-iperpiano:

$(a_1, \dots, a_n) \sim (l_1, \dots, l_n)$, cioè

$\exists \lambda \neq 0$ tale che $(a_1, \dots, a_n) = \lambda \cdot (l_1, \dots, l_n)$

Nel nostro caso $P_\infty \equiv (0, l_1, \dots, l_n)$ è polo di un iperpiano principale $\iff \exists \lambda \neq 0$ tale che

$$M^0_0 \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

20/3/09 es 16 Trovare i piani princ. di

$Q: x^2+y^2+9z^2-4xy+z=0$

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

$$M_0^0 = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

pol. car.:

$\lambda^3 - 11\lambda^2 + 15\lambda + 27 =$

$= (\lambda-3)(\lambda+1)(\lambda-9)$

$$\begin{pmatrix} (\lambda-1) & 2 & 0 \\ 2 & (\lambda-1) & 0 \\ 0 & 0 & (\lambda-9) \end{pmatrix}$$

$$\lambda=3 \quad \begin{pmatrix} \cancel{2} & \cancel{2} & \cancel{0} \\ 2 & 2 & 0 \\ 0 & 0 & -6 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \\ 0 \end{pmatrix}$$

$\begin{cases} 2l+2m=0 \\ -6n=0 \end{cases}$ $\begin{cases} l=-m \\ n=0 \end{cases}$ $P_{1\infty} = (0,-1,1,0)$

piano polare di $P_{1\infty}$:

$$(0\ -1\ 1\ 0)\, A \cdot \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$-3x+3y=0$

$\lambda=-1$

$\lambda=9$