$$\begin{pmatrix} k & 0 & \\ 0 & 1 & \\ & & 1 \end{pmatrix}$$

$x^2+y^2+z^2+k=0$

$k=0 \quad x^2+y^2+z^2=0$

$(0,0,0)$

$$\begin{pmatrix} 1 & 0 & \\ 0 & 1 & \\ & & k \end{pmatrix}$$

$x^2+y^2+kz^2+1=0$

$k=0 \quad x^2+y^2+1=0$

$$\begin{cases} X_1^2+X_2^2+X_0^2=0 \\ X_0=0 \end{cases}$$

$$(0,0,0,1) \begin{cases} X_1^2+X_2^2=0 \\ X_0=0 \end{cases}$$

Data la conica $\Gamma$ : $x^2-6xy-7y^2+4x-1=0$

a) classificarla, b) trovarne gli asintoti, c) trovarne gli assi, verificare iperbole d) trovarne il centro.

a) $A=\begin{pmatrix}-1 & 2 & 0\\ 2 & 1 & -3\\ 0 & -3 & -7\end{pmatrix}$ $|A|=44\neq 0$

$M_0^0=\begin{pmatrix}1 & -3\\ -3 & -7\end{pmatrix}$ $|M_0^0|=-16<0$ iperbole

b) $\begin{cases} X_1^2-6X_1X_2-7X_2^2+\cancel{4X_1X_0}-\cancel{X_0^2}=0\\ X_0=0\end{cases}$ $\begin{cases} X_1^2-6X_1X_2-7X_2^2=0\\ X_0=0\end{cases}$

$\begin{cases} X_1=3X_2\pm\sqrt{9X_2^2+7X_2^2}=3X_2\pm4X_2 \begin{cases} 7X_2 \\ -X_2\end{cases}\\ X_0=0\end{cases}$

$P_{1\infty}=(0,7,1)$

$P_{2\infty}=(0,1,1)$

$a_1$: $(0\ 7\ 1)\cdot\begin{pmatrix}-1 & 2 & 0\\ 2 & 1 & -3\\ 0 & -3 & -7\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0$ $14+4x-28y=0$

$a_2$: $(0\ -1\ 1)$ $-2-4x-4y=0$

$$\begin{cases} 14+4x-28y=0 \\ -2-4x-4y=0 \end{cases} \quad \begin{cases} 2x-14y=-7 \\ 2x+2y=-1 \end{cases} \quad \begin{cases} 2x-14y=-7 \\ 16y=6 \end{cases} \quad \begin{cases} 2x=-7+14\cdot\frac{3}{8}= \\ y=\frac{3}{8} \end{cases}$$

$$\begin{cases} x=-\frac{14}{16}=-\frac{7}{8} \\ y=\frac{3}{8} \end{cases} \quad d$$

c) $M^{\circ}_{d}=\begin{pmatrix} 1 & -3 \\ -3 & -7 \end{pmatrix}$ pol. car.: $\begin{pmatrix} (\lambda-1) & 3 \\ 3 & (\lambda+7) \end{pmatrix}$ $\lambda^2+6\lambda-16=$

$=(\lambda+8)(\lambda-2)$

$\lambda=-8$ $\begin{pmatrix} \cancel{9} & \cancel{3} \\ 3 & -1 \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix}=\begin{pmatrix} \cancel{0} \\ 0 \end{pmatrix}$ autospazio $U_{-8}$: $3l-m=0$ $m=3l$ $(l,m)\sim(1,3)$

polo di un asse $Q_{1\infty}\equiv(0,1,3)$

$\lambda=2$ $\begin{pmatrix} 1 & 3 \\ \cancel{3} & \cancel{9} \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix}=\begin{pmatrix} 0 \\ \cancel{0} \end{pmatrix}$ $U_2$: $l+3m=0$ $l=-3m$ $(l,m)\sim(-3,1)$

polo di un asse $Q_{2\infty}\equiv(0,-3,1)$

asse1: $(0\ 1\ 3)$ $\begin{pmatrix} -1 & 2 & 0 \\ 2 & 1 & -3 \\ 0 & -3 & -7 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \end{pmatrix}=0$ $\quad 2-8x-24y=0 \quad 4x+12y=1$

asse2: $(0\ -3\ 1)$ $\quad -6-6x+2y=0 \quad 3x-y=3$

Se non ho né assi né asintoti posso, per esempio, intersecare due diametri "comodi":

$X_\infty \equiv (0,1,0)$ polare di $X_\infty$: $(0\ 1\ 0)\begin{pmatrix}-1 & 2 & 0\\ 2 & 1 & -3\\ 0 & -3 & -7\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0$ $\left\{\begin{array}{l} 2+x-3y=0 \\ -3x-7y=0 \end{array}\right.$

$Y_\infty \equiv (0,0,1)$ " " $Y_\infty$: $(0\ 0\ 1)$

---

Data $\Gamma: x^2-6xy+9y^2-2y+1=0$ a) Verificare che è una parabola b) trovarne l'asse c) trovarne il centro

a) $A=\begin{pmatrix}1 & 0 & -1\\ 0 & 1 & -3\\ -1 & -3 & 9\end{pmatrix}$ $|A|=-1$ $A^0_0=\begin{vmatrix}1 & -3\\ -3 & 9\end{vmatrix}=0$ parabola

b) $\begin{pmatrix}\lambda-1 & 3\\ 3 & \lambda-9\end{pmatrix}$ pol. car.: $(\lambda-1)(\lambda-9)-9=\lambda^2-\lambda-9\lambda+9-9=\lambda^2-10\lambda=\cancel{\lambda}(\lambda-10)$

$\lambda=10$ $\begin{pmatrix}\cancel{9} & \cancel{3}\\ 3 & 1\end{pmatrix}\begin{pmatrix}l\\ m\end{pmatrix}=\begin{pmatrix}\cancel{0}\\ 0\end{pmatrix}$ autospazio $U_{10}$: $3l+m=0$ $m=-3l$ $(l,m)\sim(1,-3)$

polo dell'asse: $S_\infty\equiv(0,1,-3)$

asse: $(0\ 1\ -3)\begin{pmatrix}1 & 0 & -1\\ 0 & 1 & -3\\ -1 & -3 & 9\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0$ $3+10x-30y=0$

c) centro: punto improprio dell'asse: $A_\infty\equiv(0,30,10)\sim(0,3,1)$

altro calcolo del centro:

$x^2-6xy+9y^2-2y+1=0$

$$\begin{cases} X_1^2-6X_1X_2+9X_2^2-\cancel{2X_2X_0}+\cancel{X_0^2}=0 \\ X_0=0 \end{cases}$$

$$\begin{cases} (X_1-3X_2)^2=0 \\ X_0=0 \end{cases}$$

un unico punto con molteplicità 2:

$$\begin{cases} X_1=3X_2 \\ X_0=0 \end{cases} \quad (0,3,1)$$

altro modo: interseco l'asse col diametro polare di $X_\infty \equiv (0,1,0)$

$$(0\ 1\ 0)\cdot\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \\ -1 & -3 & 9 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} \qquad x-3y=0$$

$$\begin{cases} 3+10x-30y=0 \\ x-3y=0 \end{cases} \qquad \begin{cases} 10x-30y=-3 \\ x-3y=0 \end{cases} \qquad \left(\begin{pmatrix} 10 & -30 \\ 1 & -3 \end{pmatrix}\begin{matrix} -3 \\ 0 \end{matrix}\right)$$

rango incompl $=1$

" compl $=2$

nessuna sol.

$$\begin{cases} 3X_0+10X_1-30X_2=0 \\ X_1-3X_2=0 \end{cases}$$

$$\begin{pmatrix} 3 & 10 & -30 \\ 0 & 1 & -3 \end{pmatrix}$$

sol. $\left(\begin{vmatrix} 10 & -30 \\ 1 & -3 \end{vmatrix}, -\begin{vmatrix} 3 & -30 \\ 0 & -3 \end{vmatrix}, \begin{vmatrix} 3 & 10 \\ 0 & 1 \end{vmatrix}\right)=(0,9,3)\sim(0,3,1)$

11/9/09 1b Date le quadriche di eq.

$$\gamma x^2+\gamma y^2+\gamma z^2+2(\gamma+2)xy+2(\gamma+2)yz-2\gamma x-\gamma=0$$

trovare (se ci sono) quelle di rotazione e i loro assi di rotazione.

$$A=\begin{pmatrix} -\gamma & -\gamma & 0 & 0 \\ -\gamma & \gamma & (\gamma+2) & 0 \\ 0 & (\gamma+2) & \gamma & (\gamma+2) \\ 0 & 0 & (\gamma+2) & \gamma \end{pmatrix}$$

$$M_a^0=\begin{pmatrix} \gamma & (\gamma+2) & 0 \\ (\gamma+2) & \gamma & (\gamma+2) \\ 0 & (\gamma+2) & \gamma \end{pmatrix}$$

pol. car.:

$$\lambda^3-3\gamma\lambda^2+\{-8\gamma+\gamma^2\}\lambda+\gamma^3+8\gamma^2+8\gamma=$$

$$=(\lambda-\gamma)(\lambda^2-2\gamma\lambda-\gamma^2-8\gamma-8)$$