11/9/09 1b Date le quadriche di eq.

$$\gamma x^2+\gamma y^2+\gamma z^2+2(\gamma+2)xy+2(\gamma+2)yz-2\gamma x-\gamma=0$$

trovare (se ci sono) quelle di rotazione e i loro assi di rotazione.

$$A=\begin{pmatrix} -\gamma & -\gamma & 0 & 0 \\ -\gamma & \gamma & (\gamma+2) & 0 \\ 0 & (\gamma+2) & \gamma & (\gamma+2) \\ 0 & 0 & (\gamma+2) & \gamma \end{pmatrix}$$

$$M^0_a=\begin{pmatrix} \gamma & (\gamma+2) & 0 \\ (\gamma+2) & \gamma & (\gamma+2) \\ 0 & (\gamma+2) & \gamma \end{pmatrix}$$

pol. car.:

$$\lambda^3-3\gamma\lambda^2+\left[-8\gamma+\gamma^2\right]\lambda+\gamma^3+8\gamma^2+8\gamma=$$

$$=(\lambda-\gamma)(\lambda^2-2\gamma\lambda-\gamma^2-8\gamma-8)$$

$$=0$$

$$\lambda=\gamma\pm\sqrt{\gamma^2+\gamma^2+8\gamma+8}=$$

$$=\gamma\pm\sqrt{2}\sqrt{\gamma^2+4\gamma+4}=$$

$$=\gamma\pm\sqrt{2}\,|\gamma+2|$$

$\lambda_1 = \gamma \quad \lambda_2 = \gamma + \sqrt{2}(\gamma+2) \quad \lambda_3 = \gamma - \sqrt{2}(\gamma+2)$

$\lambda_2 = \lambda_3 \iff \gamma + 2 = 0 \iff \gamma = -2$

$\lambda_1 = \lambda_2 \iff \gamma = \gamma + \sqrt{2}(\gamma+2) \iff \gamma = -2$

$\lambda_1 = \lambda_3 \iff \gamma = \gamma - \sqrt{2}(\gamma+2) \iff \gamma = -2$

$\gamma = -2$

$$M_0 = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad A \cdot \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$\left\{ \begin{array}{l} = \\ = \\ = \end{array} \right.$$

Parab. iperb. $\begin{pmatrix} a & & \\ & -b & \\ & & 0 \end{pmatrix}$ $a, b > 0$ $c > 0$

Parab. ellittico $\begin{pmatrix} a & & \\ & b & \\ & & 0 \end{pmatrix}$

Iperbol. iperbolico ellittico $\begin{pmatrix} a & & \\ & b & \\ & & -c \end{pmatrix}$

Ellissoide (reale) $\begin{pmatrix} a & & \\ & b & \\ & & c \end{pmatrix}$

$C_\infty = Im[f] \cap \Pi_\infty$

Π_∞

Ellissoide

Taglio con un piano π

$r_\infty = \pi \cap \Pi_\infty$

m'interessa

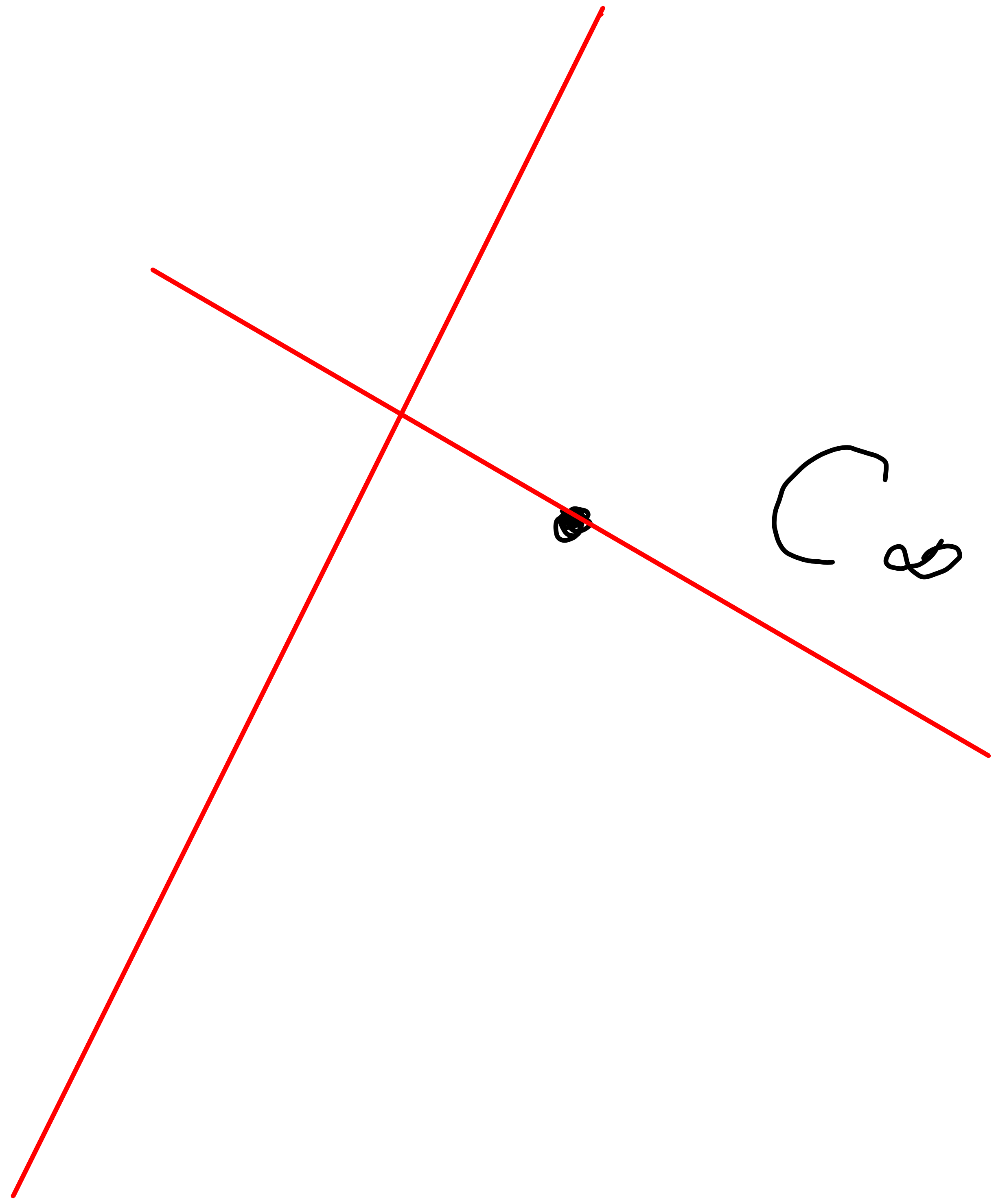
$Im[f] \cap \pi \cap \Pi_\infty$

$= Im[f] \cap r_\infty$

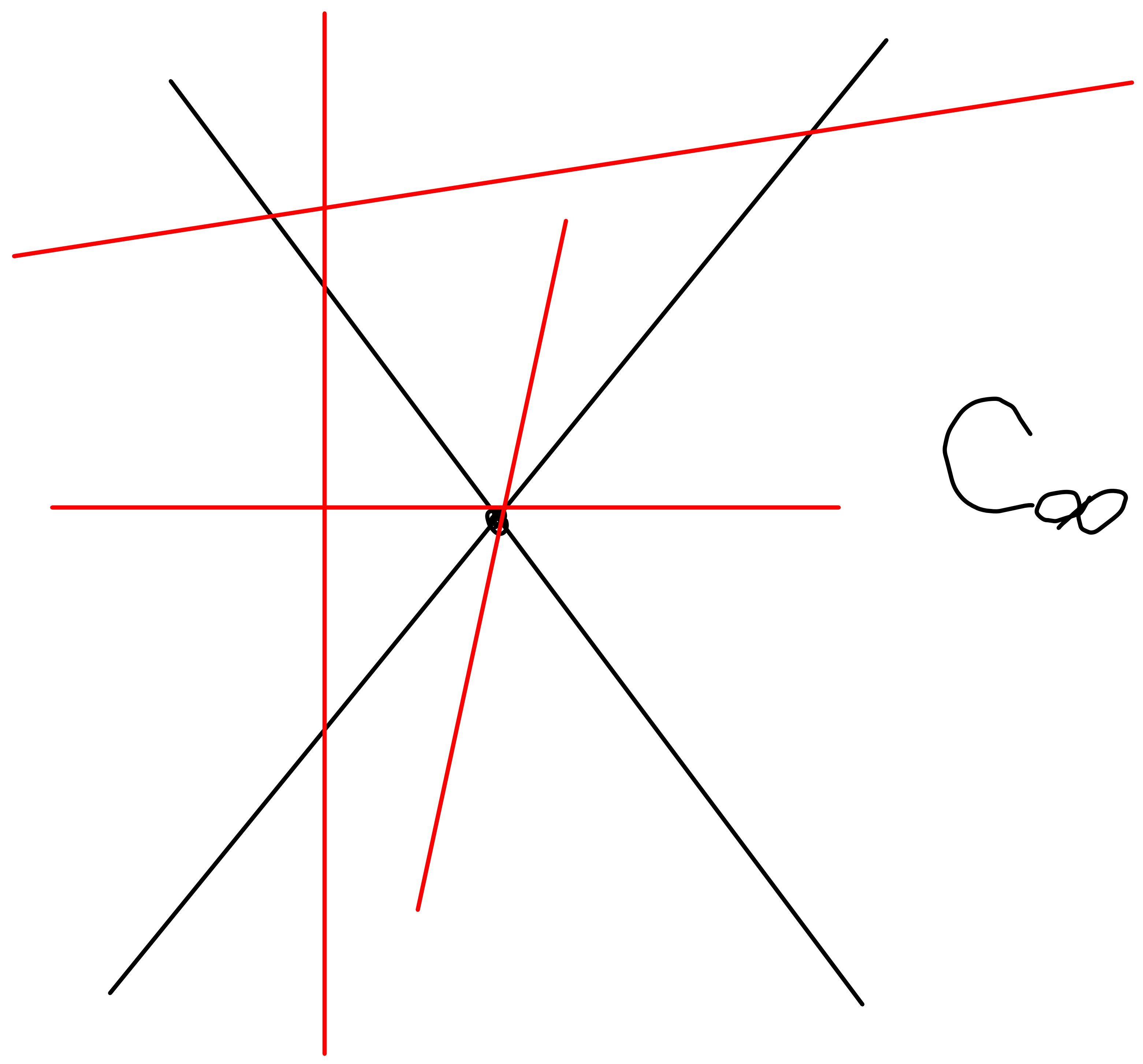
r_∞

$C_\infty = \emptyset$

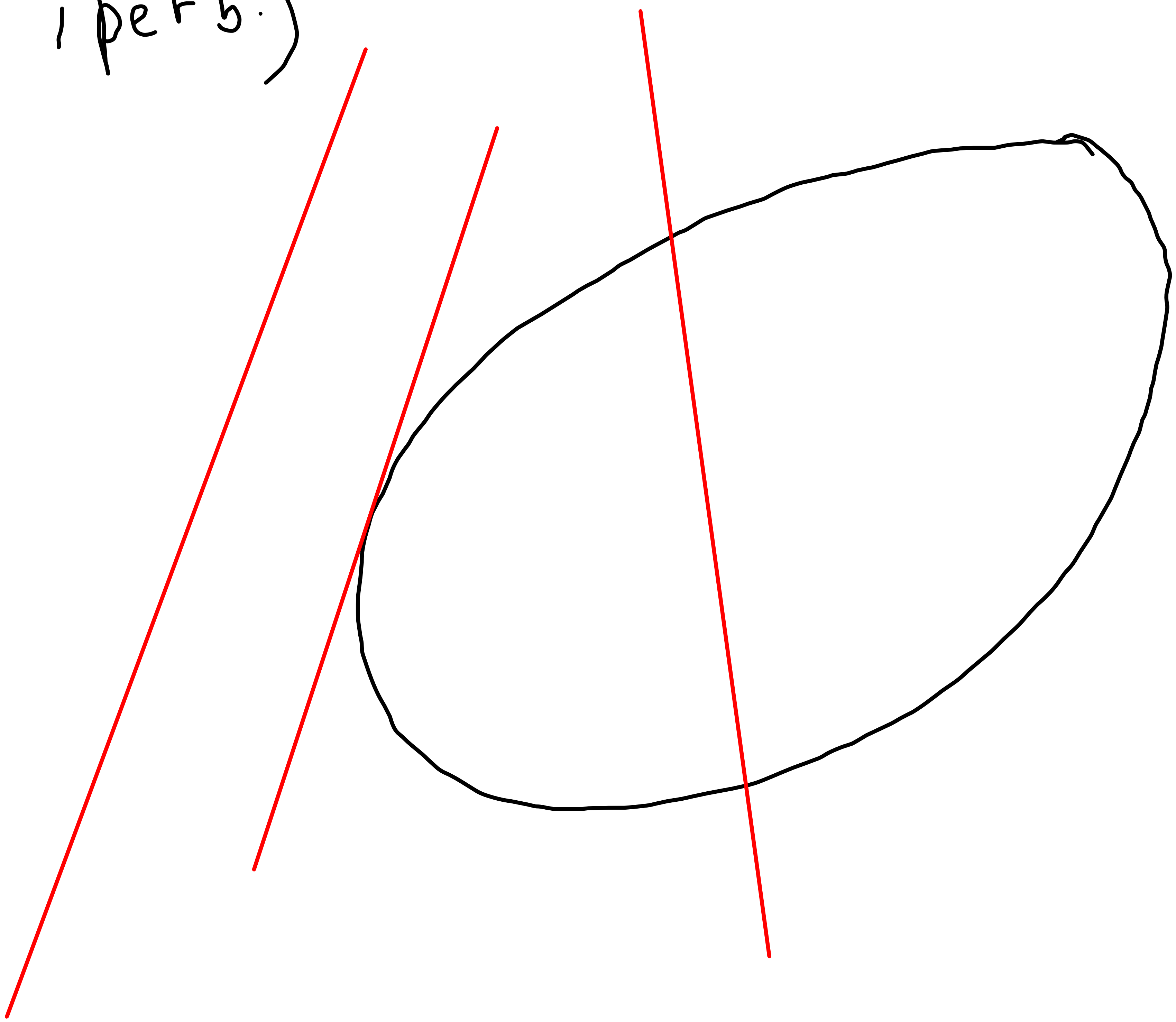
Paraboloide ellittico



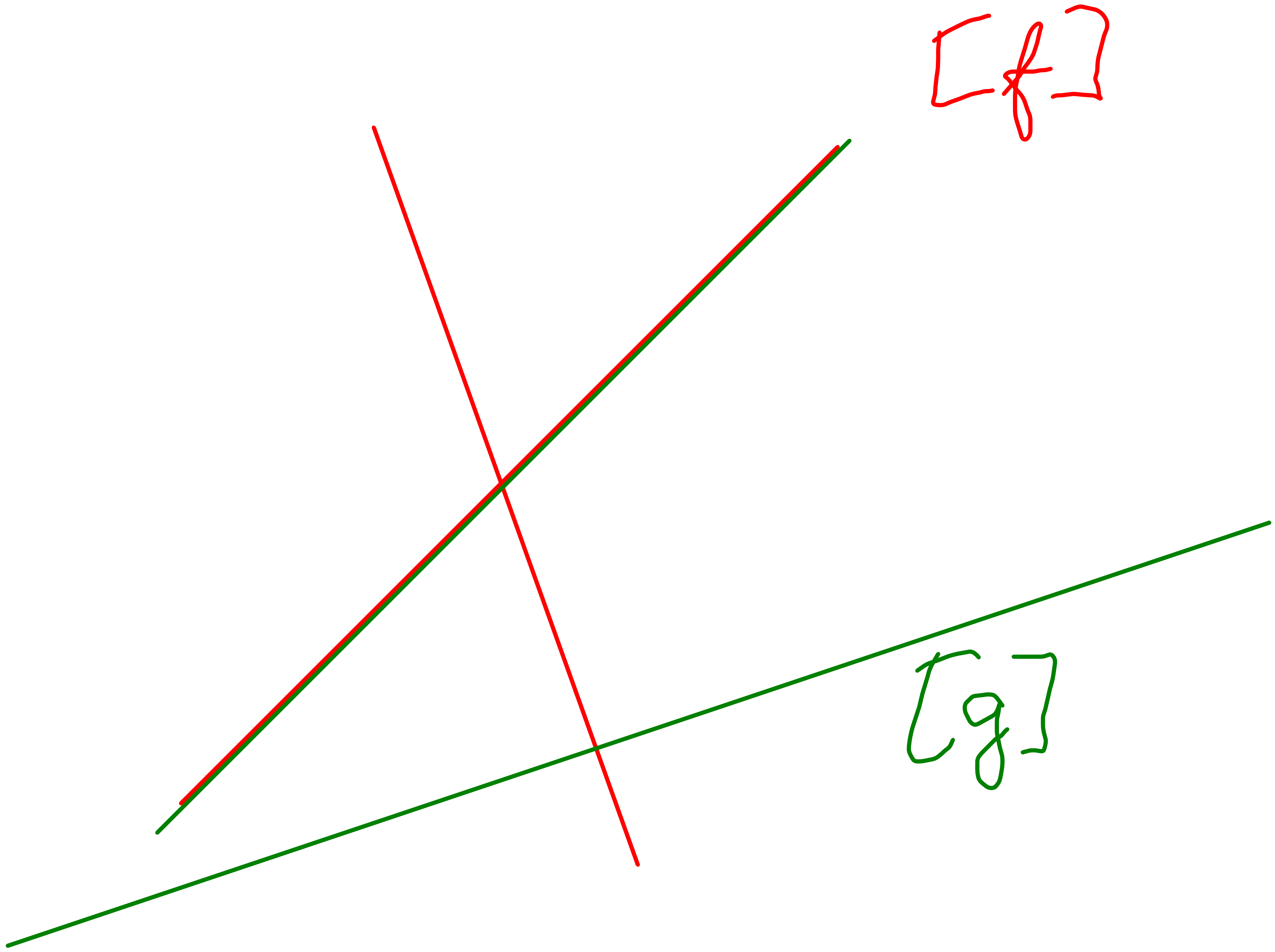
Paraboloide iperbolico

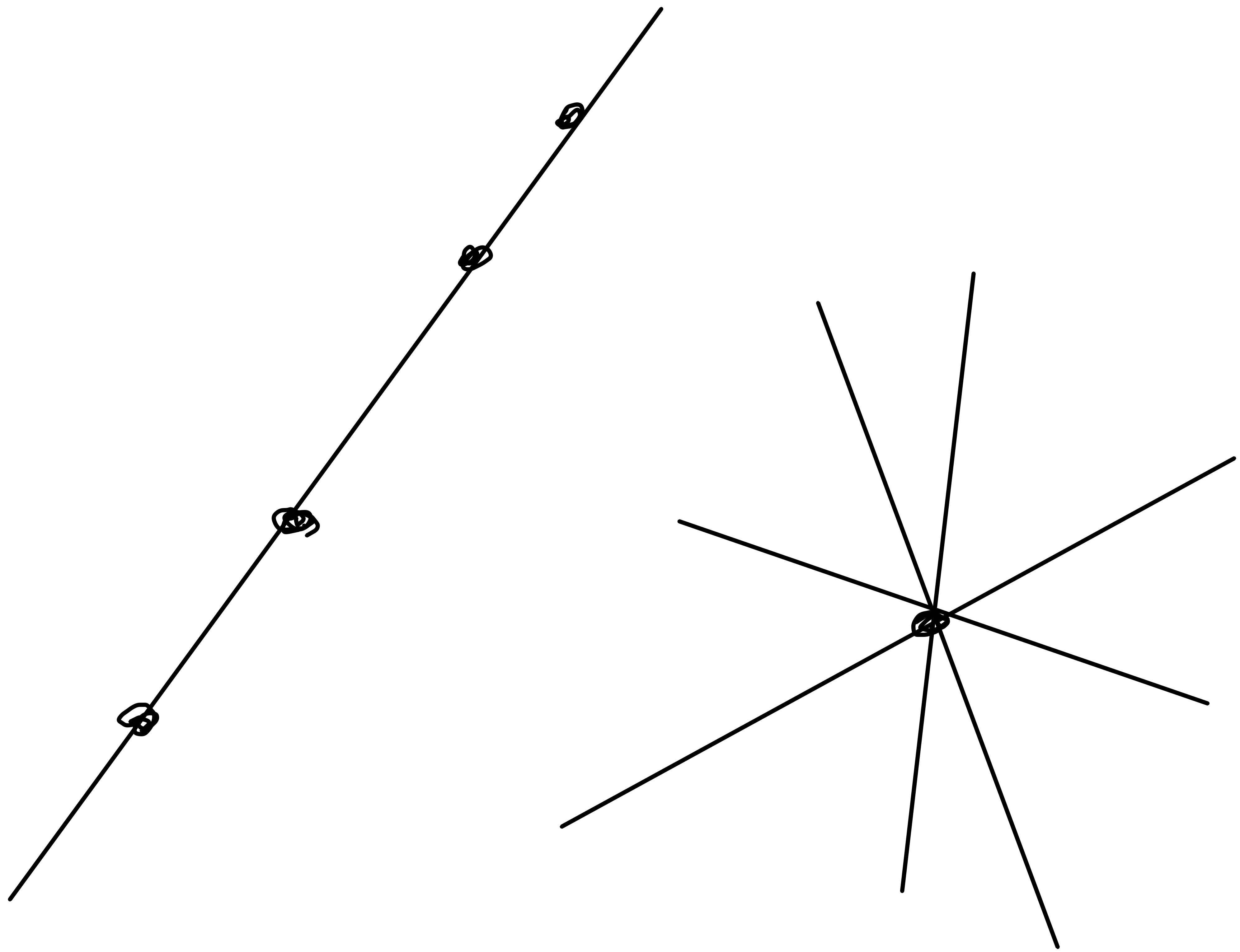


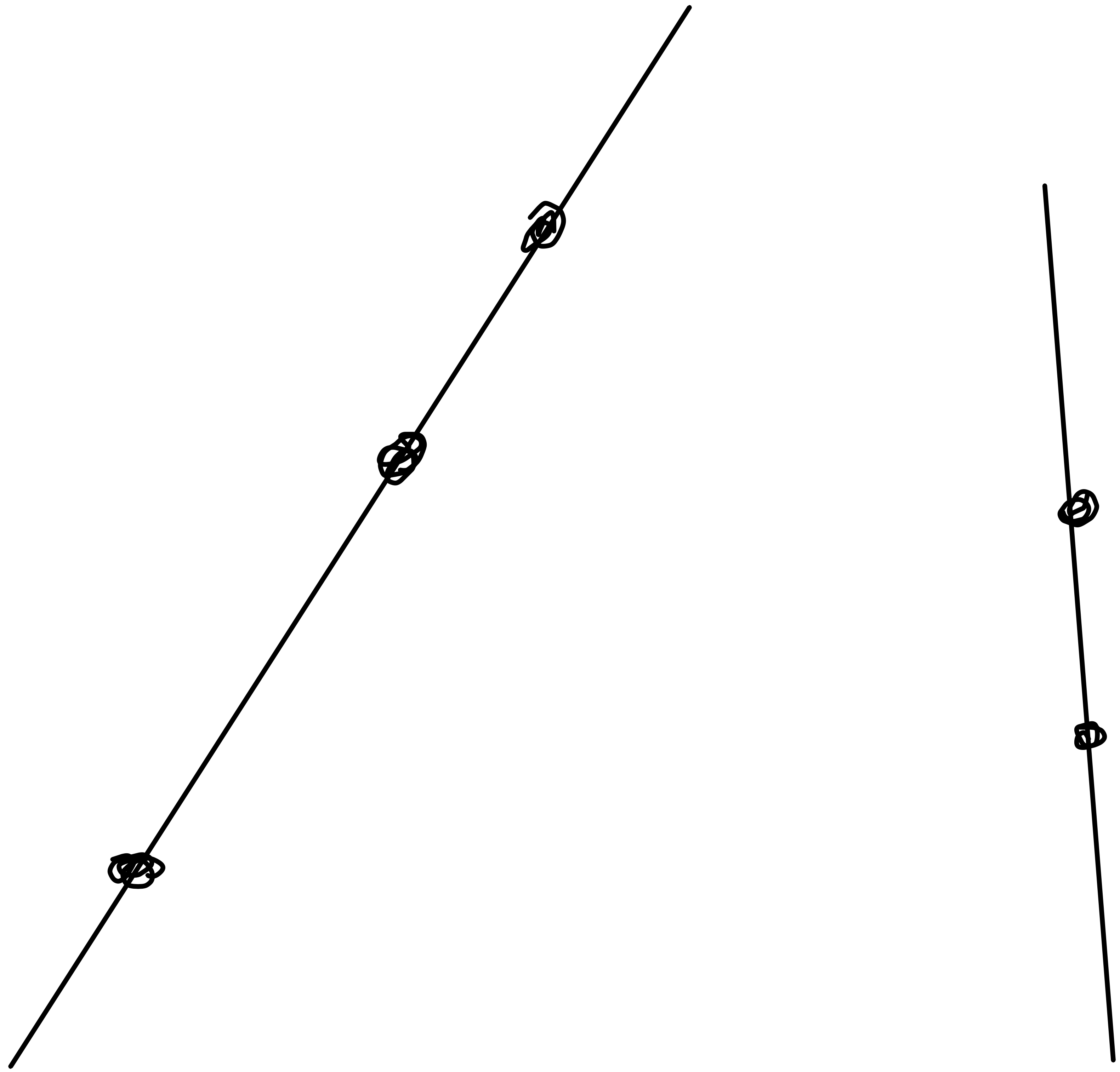
Iperboloide
(ell. o iperb.)



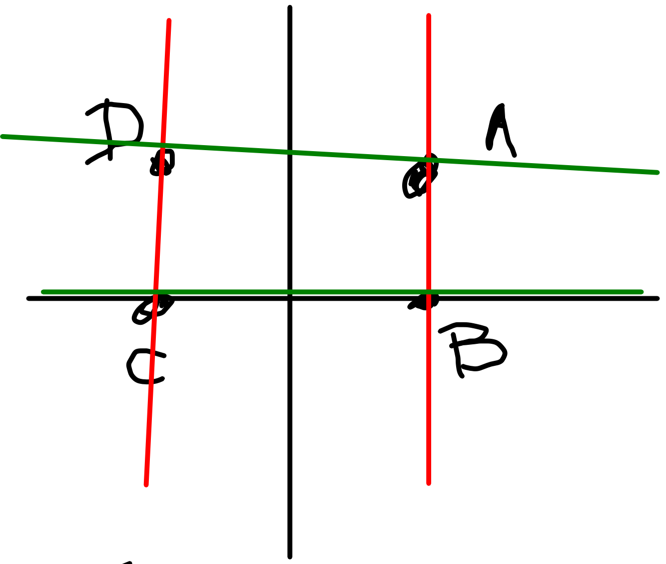
C_∞


[f]
[g]





$$
\begin{cases}
a_{00}\bar{X}_0^2 + 2a_{01}\bar{X}_0\bar{X}_1 \cdots = 0 \\
a_{00}X_0'^2 + \cdots = 0 \\
a_{00}\tilde{X}_0^2 + \cdots = 0 \\
a_{00}X_0''^2 + \cdots = 0 \\
a_{00}X_0^{\#2} + \cdots = 0
\end{cases}
$$



Dati $A \equiv (1,1)$, $B \equiv (1,0)$, $C \equiv (-1,0)$, $D \equiv (-1,1)$ si trovi il fascio di coniche per essi

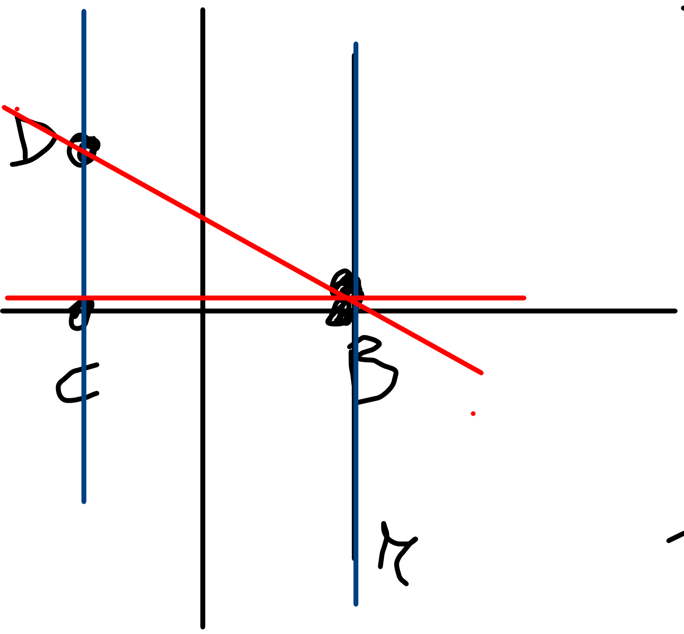
$\Gamma_1 = AB \cup CD$ $AB: x-1=0$

$\Gamma_2 = AD \cup BC$ $CD: x+1=0$

$AD: y-1=0$

$BC: y=0$

$\mathcal{F}: \lambda(x-1)(x+1)+\mu(y-1)y=0$



Dati $B \equiv (1,0)$, $C \equiv (-1,0)$, $D \equiv (-1,1)$ ed $r: x-1=0$, trovare il fascio di coniche passanti per C e D e tangenti in B ad r.

$CD: x+1=0$

$BC: y=0$

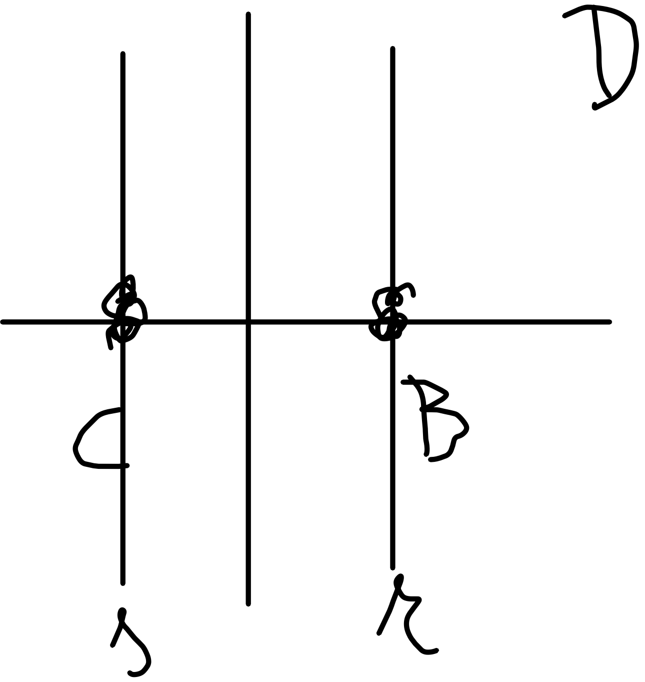
$\Gamma_1 = r \cup CD$

$\Gamma_2 = BC \cup BD$

$BD: \dfrac{x-1}{-1-1} = \dfrac{y-0}{1-0}$

$\dfrac{x-1}{-2} = y \qquad x+2y-1=0$

$\mathcal{F}: \lambda(x-1)(x+1) + \mu\, y(x+2y-1) = 0$



Dati $B \equiv (1,0)$, $C \equiv (-1,0)$,

$r: x = 1$, $s: x = -1$

Trovare il fascio di coniche tangenti in B ad r e in C ad s.

$BC: y = 0$

$\Gamma_1 = r \cup s$

$\Gamma_2 = BC$ contata 2 volte

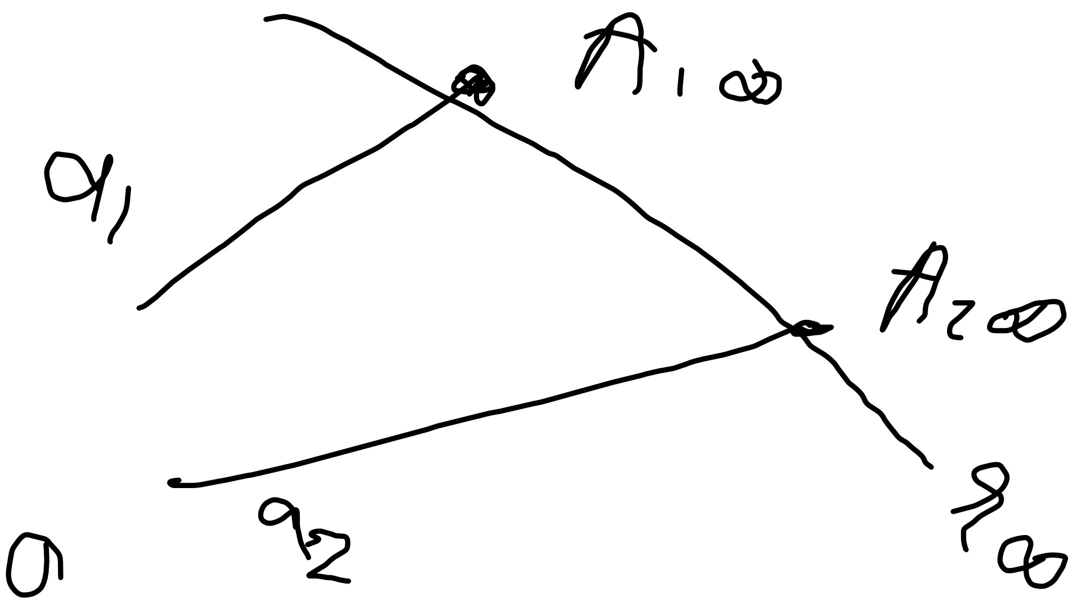
$$\mathcal{F}: \lambda(x-1)(x+1) + \mu y^2 = 0$$

Trovare il fascio di iperboli aventi asintoti

$a_1 : x - y = 0$, $a_2 : y - 2x - 1 = 0$

$X_1 - X_2 = 0$ $\qquad$ $X_2 - 2X_1 - X_0 = 0$

$A_{1\infty} \equiv (0, 1, 1)$ $\qquad$ $A_{2\infty} \equiv (0, 2, 1)$



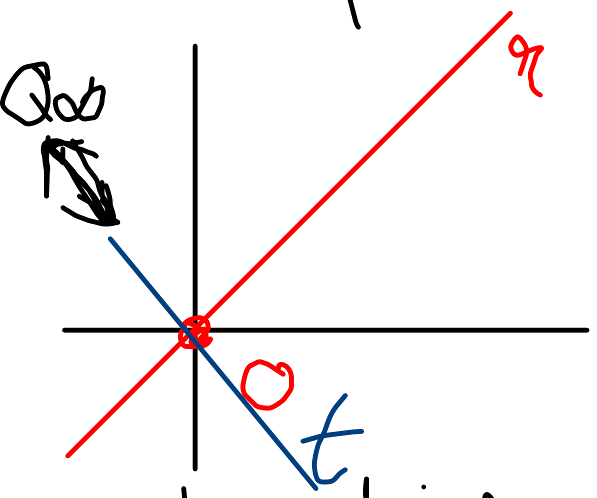
$\Gamma_1 = a_1 \cup a_2$

$\Gamma_2 = r_\infty$ contata 2 volte

$$\mathcal{F} : \lambda (X_1 - X_2)(X_2 - 2X_1 - X_0) + \mu X_0^2 = 0$$

$$\lambda (x - y)(y - 2x - 1) + \mu = 0$$

Data la retta $r: y=x$ ed $O\equiv(0,0)$, si trovi il fascio di parabole aventi asse r e vertice O,



$R_\infty \equiv (0,1,1)$ punto di tangenza con r_∞

O

Polo di r: $Q_\infty\equiv(0,1,-1)$

Polare di Q_∞ (r) passa per O $\Rightarrow$ polare di O passa per Q_∞ (tang. in O)

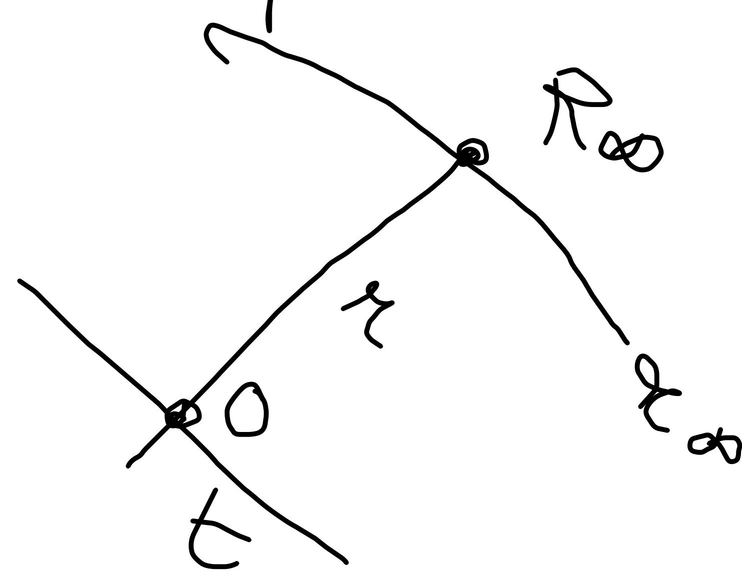
Tang. in O: $t: \dfrac{x-0}{1}=\dfrac{y-0}{-1}$ $\quad x+y=0$

$\Gamma_1 = t \cup r_\infty$

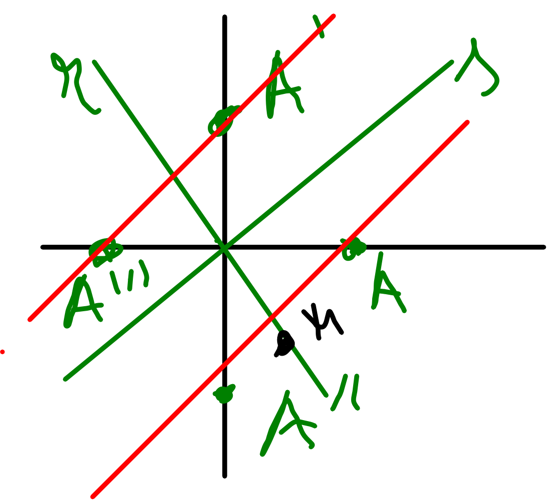
$\Gamma_2 = r$ contata 2 volte

$\mathcal{F}: \lambda(X_1-X_2)^2+\mu(X_1+X_2)X_0=0$

$\lambda(x-y)^2+\mu(x+y)=0$



Siano $r: x+y=0$, $s: x-y=0$, $A \equiv (1,0)$.
Si trovi il fascio di coniche passanti per A e aventi r ed s come assi.



$A' \equiv (0,1)$, $A'' \equiv (0,-1)$, $A''' \equiv (-1,0)$

Come trovo A''?

AA''

Retta per $A \perp r$: $\dfrac{x-1}{1} = \dfrac{y-0}{1}$ $\quad x-y-1=0$

Interseco con r: $\begin{cases} x+y=0 \\ x-y=1 \end{cases}$ $\begin{cases} x+y=0 \\ -2y=1 \end{cases}$ $\begin{cases} x=-y=\frac{1}{2} \\ y=-\frac{1}{2} \end{cases}$

$M \equiv \left(\frac{1}{2}, -\frac{1}{2}\right)$

$\dfrac{A+A''}{2} = M$ $\quad A'' = 2M - A = 2\left(\frac{1}{2}, -\frac{1}{2}\right) - (1,0) =$

$= (1,-1) - (1,0) = (0,-1)$

$\Gamma_1 = \overset{y=0}{A'''A} \cup \overset{x=0}{A'A''}$ $\quad \mathcal{F}: \lambda xy + \mu(x-y+1)(x-y-1) = 0$

$\Gamma_2 = \underset{x-y-1=0}{A''A} \cup \underset{x-y+1=0}{A'A'''}$

Si trovi la parabola avente asse $r: y = x$, avente vertice $O \equiv (0,0)$ e passante per $P \equiv (1,2)$

Costruisco il fascio e cerco la conica del fascio che rispetti

$$\mathcal{F}: \lambda (x-y)^2 + \mu (x+y) = 0$$

$$\lambda (1-2)^2 + \mu (1+2) = 0$$

$$\lambda + 3\mu = 0 \qquad \lambda = -3\mu$$

$$-3(x-y)^2 + (x+y) = 0$$

$$-3x^2 - 3y^2 + 6xy + x + y = 0$$

Es 17 Date $r: y=0$ $s: y=2x$

$\Gamma: 2xy-y^2-2x+3y+4=0$

Trovare il fascio di coniche passanti per i punti di intersezione di Γ con r ed s.

$\mathcal{F}: \lambda(2xy-y^2-2x+3y+4)+\mu y(y-2x)=0$

r di coeff. dir. $(l_1, \dots, l_n)$, r' di coeff. dir $(l'_1, \dots, l'_n)$
π di eq. $a_1x_1 + \dots + a_nx_n = b$, π' di eq. $a'_1x_1 + \dots + a'_nx_n = b'$

risp. a un rif. affine	risp. a un rif. cart.
$r \parallel r' \iff (l_1, \dots, l_n) \sim (l'_1, \dots, l'_n)$	$r \perp r' \iff l_1l'_1 + \dots + l_nl'_n = 0$
$r \parallel \pi \iff a_1l_1 + \dots + a_nl_n = 0$	$r \perp \pi \iff (a_1, \dots, a_n) \sim (l_1, \dots, l_n)$
$\pi \parallel \pi' \iff (a_1, \dots, a_n) \sim (a'_1, \dots, a'_n)$	$\pi \perp \pi' \iff a_1a'_1 + \dots + a_na'_n = 0$

$r: \begin{cases} (a+ib)x + (c+id)y + e+if = 0 \end{cases}$ retta imm.

considero la coniugata

$\bar{r}: (a-ib)x + (c-id)y + e-if = 0$

$r \cap \bar{r}: \begin{cases} 2ax + 2cy + 2e = 0 \\ 2\cancel{i}bx + 2\cancel{i}dy + 2\cancel{i}f = 0 \end{cases}$ 2 rette reali

$\Longrightarrow$ 1 punto reale