Similitudine: $A = E^{-1} \cdot A' \cdot E$

$$|A| = |E^{-1}| \cdot |A'| \cdot |E| = |A'| \cdot \cancel{|E^{-1}|} \cdot \cancel{|E|}$$

Congruenza: $A = {}^tE \cdot A' \cdot E$

$$|A| = |{}^tE| \cdot |A'| \cdot |E| = |A'| \cdot |{}^tE| \cdot |E| = |A'| \cdot (|E|)^2$$

$\begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \begin{pmatrix} -1 & \\ & 0 \end{pmatrix}$	det = 0	$\begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} \begin{pmatrix} -1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$	det = 0
$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} -1 & \\ & -1 \end{pmatrix}$	det > 0	$\begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \begin{pmatrix} -1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$	det = 0
$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$	det < 0	$\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$	det = 0

$\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$ $\det > 0$

$\begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$ $\det < 0$

$\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$ $\det < 0$

$\begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$ $\det > 0$

$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$ $\det > 0$

$\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ $\det > 0$

$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$ $\det < 0$

$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ $\det < 0$

$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$ $\det > 0$

Classificare, al variare di $\gamma \in \mathbb{R}$, le coniche

$x^2+2\gamma xy+y^2-10=0 \qquad X_1^2+2\gamma X_1X_2+X_2^2-10X_0^2=0$

$$A_\gamma=\begin{pmatrix}-10&0&0\\0&1&\gamma\\0&\gamma&1\end{pmatrix} \qquad |A_\gamma|=-10\begin{vmatrix}1&\gamma\\\gamma&1\end{vmatrix}=-10(1-\gamma^2)=10(\gamma^2-1)$$

$-1 \qquad +1$

deg

M_0^0

$$A_0^0=|M_0^0|=1-\gamma^2 \;\begin{matrix}>\\=\\<\end{matrix}\; 0 \iff \begin{matrix}-1<\gamma<1\\ \gamma=\pm1\\ \gamma<-1 \vee \gamma>1\end{matrix}$$

	$\lvert A\rvert$	rkA	A_0^0	Coniche
$\gamma<-1$	$\neq 0$	3	$-$	iperboli
$\gamma=-1$	$=0$	2	0	deg $rk=2$
$-1<\gamma<1$	$\neq 0$	3	$+$	ellissi reali (discordanza sulla diag. princ.)
$\gamma=1$	$=0$	2	0	deg $rk=2$
$\gamma>1$	$\neq 0$	3	$-$	iperboli

$\gamma=-1$

$$\begin{pmatrix}-10&0&0\\0&1&-1\\0&-1&1\end{pmatrix}$$

$\gamma=1$

$$\begin{pmatrix}-10&0&0\\0&1&1\\0&1&1\end{pmatrix}$$

Idem per: $(\gamma-1)x^2+20xy+(\gamma-1)y^2+2\gamma x=0$

$$A_\gamma = \begin{pmatrix} 0 & \gamma & 0 \\ \gamma & (\gamma-1) & 10 \\ 0 & 10 & (\gamma-1) \end{pmatrix} \qquad |A_\gamma| = -\gamma \begin{vmatrix} \gamma & 10 \\ 0 & (\gamma-1) \end{vmatrix} = -\gamma^2(\gamma-1)$$

		0		1	
$-\gamma^2$	$-$	0	$-$		$-$
$(\gamma-1)$	$-$		$-$	0	$+$
$\lvert A_\gamma\rvert$	>0	0	>0	0	<0
		deg.		deg.	

$$A_a^o = |M_a^a| = \begin{vmatrix} (\gamma-1) & 10 \\ 10 & (\gamma-1) \end{vmatrix} = (\gamma-1)^2-10^2 = (\gamma-1+10)(\gamma-1-10) =$$

$$= (\gamma+9)(\gamma-11) > 0 \iff \gamma<-9 \vee \gamma>11$$

	$\lvert A\rvert$	rkA	A_o^o	coniche
$\gamma<-9$	$\neq 0$	3	+	ellissi reali *
$\gamma=-9$	$\neq 0$	3	0	parabola
$-9<\gamma<0$	$\neq 0$	3	−	iperboli
$\gamma=0$	$=0$	2	−	deg rk = 2
$0<\gamma<1$	$\neq 0$	3	−	iperboli
$\gamma=1$	$=0$	2	−	deg rk=0
$1<\gamma<11$	$\neq 0$	3	−	iperboli
$\gamma=11$	$\neq 0$	3	0	parabola
$\gamma>11$	$\neq 0$	3	+	ellissi reali *

*(0 sulla diag. princ.)

$\gamma=0$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 10 \\ 0 & 10 & -1 \end{pmatrix}$$

$\gamma=1$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 10 \\ 0 & 10 & 0 \end{pmatrix}$$

Classificare le quadriche $11x^2+(\gamma+1)y^2+11z^2+2\gamma y-18xz=0$

$$11X_1^2+(\gamma+1)X_2^2+11X_3^2+2\gamma X_2X_0-18X_1X_3=0$$

$$A_\gamma=\begin{pmatrix}0&0&\gamma&0\\0&11&0&-9\\\gamma&0&(\gamma+1)&0\\0&-9&0&11\end{pmatrix}$$

$$|A_\gamma|=\gamma\begin{vmatrix}0&11&-9\\\gamma&0&0\\0&-9&11\end{vmatrix}=-\gamma^2\begin{vmatrix}11&-9\\-9&11\end{vmatrix}=-40\gamma^2$$

$$M_0^0=\begin{pmatrix}11&0&-9\\0&(\gamma+1)&0\\-9&0&11\end{pmatrix}\qquad A_0^0=|M_0^0|=(\gamma+1)\begin{vmatrix}11&-9\\-9&11\end{vmatrix}=40(\gamma+1)$$

0

ellittiche · deg · ellittiche

-1 · 0

$|M_3|$

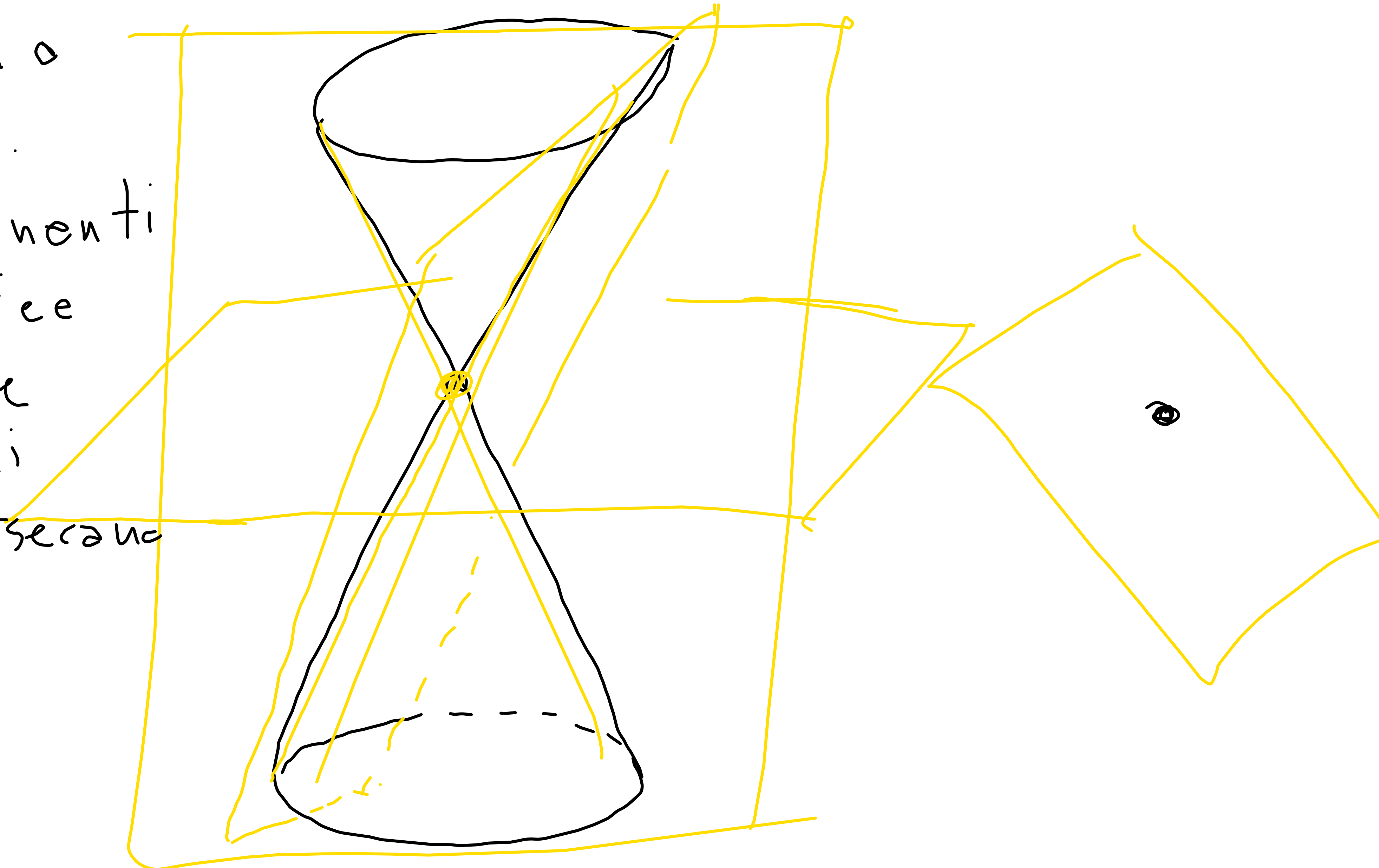
$|M_2|=\begin{vmatrix}11&0\\0&(\gamma+1)\end{vmatrix}=11(\gamma+1)$

$|M_1|=|11|$

±	0	+ + +
ind	ind	d. pos.

	$\lvert A\rvert$	rk A	M_0^0	quadriche
$\gamma<-1$	−	4	ind.	iperboloidi ellittici
$\gamma=-1$	−	4	deg.	paraboloide ellittico
$-1<\gamma<0$	−	4	d. pos.	ellissoidi reali
$\gamma=0$	0	3	d. pos.	cono imm.
$\gamma>0$	−	4	d. pos.	ellissoidi reali

PROP-
Dato un cono proiettivo, i piani contenenti il suo vertice sono tutti e soli i piani che lo intersecano in coniche degeneri.



[f] iperq. non spec., con discriminante

$$A = \begin{pmatrix} a_0^0 & \cdots & & \\ a_0^1 & a_1^1 & & \\ & \cdots & & \end{pmatrix}$$

Cerco i poli degli iperpiani principali.

Prendo il generico punto improprio $P_\infty = (0, l_1, \ldots, l_n)$. Ne faccio l'iperpiano polare.

π: $(X_0 X_1 \cdots X_n) \cdot A \cdot \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$ $\quad (X_0 X_1 \cdots X_n) \cdot \begin{pmatrix} a_0^0 & a_1^0 & \cdots & a_n^0 \\ a_0^1 & a_1^1 & \cdots & a_n^1 \\ \vdots & & & \\ a_0^n & a_1^n & \cdots & a_n^n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$

$$(X_0 X_1 \cdots X_n) \cdot \begin{pmatrix} 0 \cdot a_0^0 + a_1^0 l_1 + \cdots + a_n^0 l_n \\ 0 \cdot a_0^1 + a_1^1 l_1 + \cdots + a_n^1 l_n \\ \vdots \\ 0 \cdot a_0^n + a_1^n l_1 + \cdots + a_n^n l_n \end{pmatrix} \begin{matrix} b_0 \\ b_1 \\ \\ b_n \end{matrix} = 0 \qquad b_0 X_0 + b_1 X_1 + \cdots + b_n X_n = 0$$

π: $b_1 x_1 + \cdots + b_n x_n + b_0 = 0$

$P_\infty \perp \pi$

$\Updownarrow$

$(b_1, \ldots, b_n) \sim (l_1, \ldots, l_n)$

$\Longleftrightarrow \exists\, \lambda \neq 0$ tale che $\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$

Cioè $M_0^0 \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$

Cerco gli assi della conica $x^2+20xy+y^2+4x=0$

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 2 & \boxed{1 \quad 10} \\ 0 & \boxed{10 \quad 1} \end{pmatrix}$$

$M^{o} =$

$$\begin{vmatrix} (\lambda-1) & -10 \\ -10 & (\lambda-1) \end{vmatrix} = (\lambda-1)^2-(-10)^2 =$$

$$=(\lambda-1-10)(\lambda-1+10) = (\lambda-11)(\lambda+9)$$

Autovalori

11

-9

$$U_{11}: \begin{pmatrix} 10 & -10 \\ \cancel{-10} & \cancel{10} \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \end{pmatrix} \qquad 10l-10m=0 \quad l=m$$

$$\mathcal{B}_{U_{11}} = ((1,1))$$

$$U_{-9}: \begin{pmatrix} -10 & -10 \\ \cancel{-10} & \cancel{-10} \end{pmatrix}\begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \end{pmatrix} \qquad -10l-10m=0 \quad l=-m$$

$$\mathcal{B}_{U_{-9}} = ((1,-1))$$

$P_\infty \equiv (0,1,1)$

$Q_\infty \equiv (0,1,-1)$

$$\begin{matrix} (0 \;\; 1 \;\; 1) \\ (0 \;\; 1 \; -1) \end{matrix} \cdot \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 10 \\ 0 & 10 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0 \qquad \begin{matrix} a_1: 2+11x+11y=0 \\ a_2: 2-9x+9y=0 \end{matrix}$$

Trovare il centro: $a_1 \cap a_2$

$$\begin{cases} x+y = -\frac{2}{11} \\ -x+y = -\frac{2}{9} \end{cases} \qquad \begin{cases} x+y = -\frac{2}{11} \\ 2y = -\frac{40}{99} \end{cases} \qquad \begin{cases} x = -\frac{2}{11} - \frac{20}{99} = -\frac{38}{99} \\ y = -\frac{20}{99} \end{cases}$$

Trovarne gli asintoti

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 10 \\ 0 & 10 & 1 \end{pmatrix}$$

$$\begin{cases} x_1^2 + 20x_1x_2 + x_2^2 + 4\cancel{x_1x_0} = 0 \\ x_0 = 0 \end{cases}$$

$$\begin{cases} x_0 = 0 \\ \frac{x_1}{x_2} = -10 \pm \sqrt{100-1} = -10+\sqrt{99} \end{cases}$$

$$Q_{1\infty} \equiv (0, -10+\sqrt{99}, 1)$$

$$Q_{1\infty} \equiv (0, -10-\sqrt{99}, 1)$$

$$\begin{pmatrix} 0 & (-10+\sqrt{99}) & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 10 \\ 0 & 10 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

$$-2+2\sqrt{99} + \sqrt{99}\,x + (-100+10\sqrt{99}+1)\,y = 0$$

as_1: $-2+2\sqrt{99}+\sqrt{99}\,x + (-99+10\sqrt{99})\,y = 0$

as_2: $-2-2\sqrt{99} - \sqrt{99}\,x + (-99-10\sqrt{99})\,y = 0$