$\Sigma: x^3 - 2y^2 - 2z - 4x + 4 = 0$ a) trovare il piano tang. Π a Σ in $P \equiv (2,0,2)$, b) Determinare la curva $\mathcal{C}'$ proiezione ortogonale, sul piano xy, della curva $\mathcal{C}$ intersezione di Σ con Π. c) Verificare che $\mathcal{C}'$ ha un punta doppia in P' proiez. ortog. di P sul piano xy.

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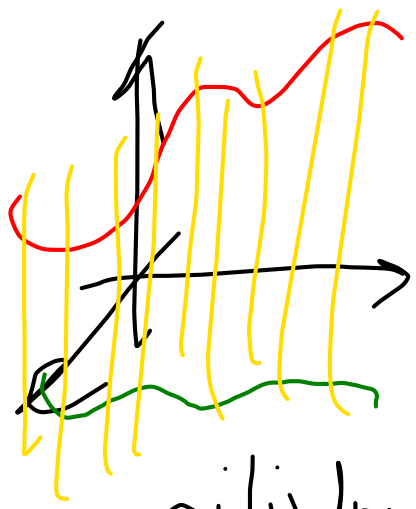
a) $\Pi: 4x - z - 6 = 0$ b) $\mathcal{C}: \begin{cases} x^3 - 2y^2 - 2z - 4x + 4 = 0 \\ 4x - z - 6 = 0 \end{cases}$



Elimino z: $\begin{cases} z = \dfrac{x^3 - 2y^2 - 4x + 4}{2} \\ z = 4x - 6 \end{cases}$

$x^3 - 2y^2 - 4x + 4 = 8x - 12$

cilindro $\rightarrow$ $x^3 - 2y^2 - 12x + 16 = 0$

$P' \equiv (2,0,0)$

$(2,0)$

$C': \begin{cases} x^3 - 2y^2 - 12x + 16 = 0 \\ z = 0 \end{cases}$ $\leftarrow$ d'ora in poi la ignoro

$$x^3 - 2y^2 - 12x + 16 = 0$$

$F(x,y)$

$$P' \equiv (2,0)$$

$F_x = 3x^2 - 12$ 0

$F_y = -4y$ 0

$F_{xx} = 6x$ 12

$F_{xy} = 0$ 0

$F_{yy} = -4$ -4

$12 - 4k^2 = 0$

$3 - k^2 = 0$ $k = \pm\sqrt{3}$

tangent in P', $(2, 0)$

$y = \sqrt{3}(x-2)$

$y = -\sqrt{3}(x-2)$

Trovare gli eventuali punti multipli (reali), e il cono tangente in uno di essi, della superficie Σ:

$x^4 + y^4 + z^4 + x^2$

$$\begin{cases} F=0 \\ x(x^2+y^2+z^2-7)=0 \\ y(x^2+y^2+z^2-7)=0 \\ z(x^2+y^2+z^2-5)=0 \end{cases}$$

① $\begin{cases} F=0 \\ x=0 \\ y=0 \\ z=0 \end{cases}$

② $\begin{cases} F=0 \\ (x^2+y^2+z^2-7)=0 \\ y=0 \\ z=0 \end{cases}$

③ $\begin{cases} F=0 \\ x=0 \\ x^2+y^2+z^2-7=0 \\ z=0 \end{cases}$

④ $\begin{cases} F=0 \\ x=0 \\ y=0 \\ x^2+y^2+z^2-5=0 \end{cases}$

⑤ $\begin{cases} F=0 \\ x^2+y^2+z^2-7=0 \\ x^2+y^2+z^2-7=0 \\ z=0 \end{cases}$

⑥ $\begin{cases} F=0 \\ x=0 \\ x^2+y^2+z^2-7=0 \\ x^2+y^2+z^2-5=0 \end{cases}$

⑦ $\begin{cases} F=0 \\ x^2+y^2+z^2-7=0 \\ y=0 \\ x^2+y^2+z^2-5=0 \end{cases}$

⑧ $\begin{cases} F=0 \\ x^2+y^2+z^2-7=0 \\ x^2+y^2+z^2-7=0 \\ x^2+y^2+z^2-5=0 \end{cases}$

① $F=0$, $x=0$, $y=0$, $z=0$ — $25=0$ — — —

$F=0$, $x^2=7$, $y=0$, $z=\infty$ — $F=0$, $x=\pm\sqrt{7}$, $y=0$, $z=0$

② $F=0$, $x^2+y^2+z^2-7=0$, $y=0$, $z=0$ — $\neq 0$ — — —

③ $F=\neq 0$, $=0$, $x=0$, $x^2+y^2+z^2-7=0$, $z=0$

$49+0+0+0+0+0-98-0-0+25\neq 0$

$x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2-14x^2-14y^2-10z^2+25=0$

④ $\begin{cases} F=0 \\ x=0 \\ y=0 \\ x^2+y^2+z^2-5=0 \end{cases}$ $\begin{cases} F=0 \\ x=0 \\ y=0 \\ z^2=5 \end{cases}$ $\begin{cases} F=0 \\ x=0 \\ y=0 \\ z=\pm\sqrt{5} \end{cases}$ $\begin{cases} 0=0 \\ \text{—} \\ \text{—} \end{cases}$ $M_1 \equiv (0,0,\sqrt{5})$ $M_2 \equiv (0,0,-\sqrt{5})$

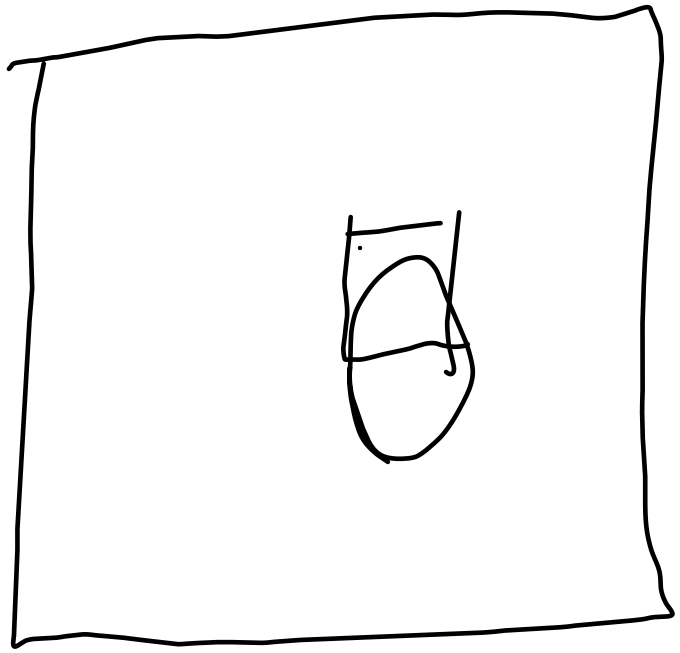
⑤ ~~$\begin{cases} F=0 \\ x^2+y^2+z^2-7=0 \\ z=0 \end{cases}$~~ ~~$\begin{cases} F=0 \\ x^2+y^2=7 \\ z=0 \end{cases}$~~ $\neq 0$

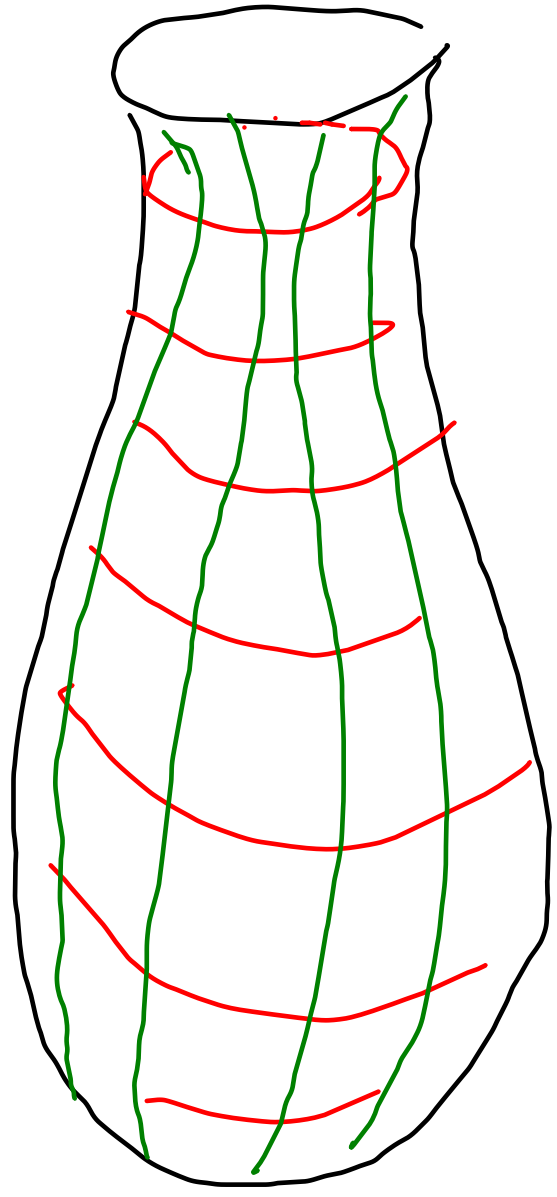
$= (x^2+y^2)^2$ $\qquad -14(x^2+y^2)$

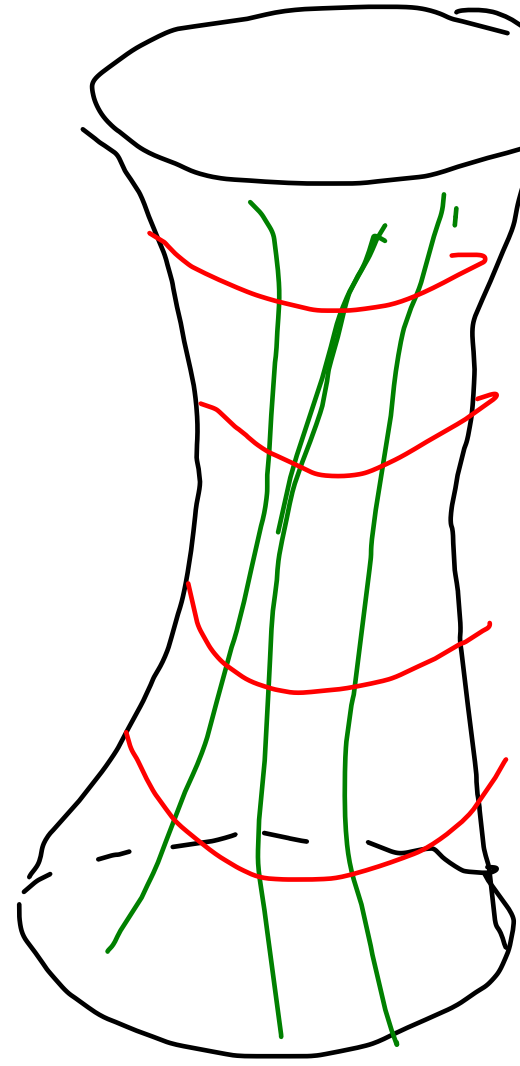
$0+0+25+0+0+0-0-0-50+25=0$

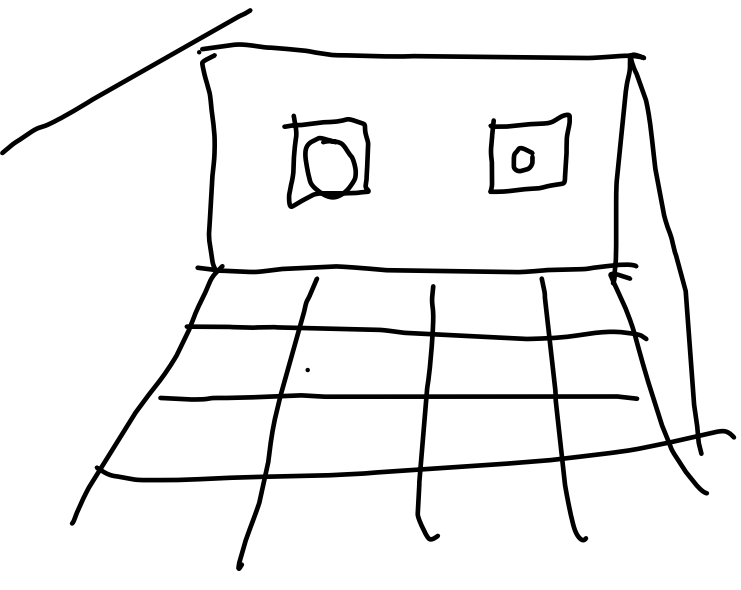
$x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2-14x^2-14y^2-10z^2+25=0$

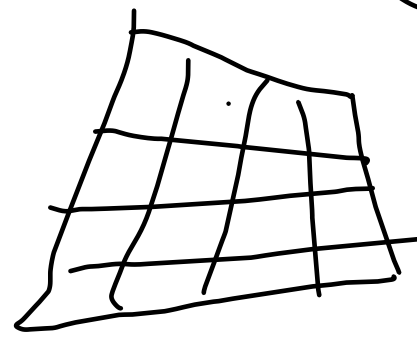
$40 \quad 0 \quad 0 \quad 0 \quad -98 \quad 0+25 \neq 0$











$$\begin{cases} F = 0 \\ x(x^2+y^2+z^2-7) = 0 \\ y(x^2+y^2+z^2-7) = 0 \\ z(x^2+y^2+z^2-5) = 0 \end{cases}$$

$F_x = 4x^3 + 4xy^2 + 4xz^2 - 28x$

$F_y = 4y^3 + 4x^2y + 4yz^2 - 28y$

$F_z = 4z^3 + 4x^2z + 4y^2z - 20z$

$F_{xx} = 12x^2 + 4y^2 + 4z^2 - 28$ -8

$F_{xy} = 8xy$ 0

$F_{xz} = 8xz$ 0

$F_{yy} = 12y^2 + 4x^2 + 4z^2 - 28$ -8

$F_{yz} = 8yz$ 0

$F_{zz} = 12z^2 + 4x^2 + 4y^2 - 20$ 40

$(0, 0, \sqrt{5})$

Cano tang

$-8x^2 + 2 \cdot 0xy + 2 \cdot 0 \cdot x(z-\sqrt{5}) - 8y^2 + 2 \cdot 0 \cdot yz + 40(z-\sqrt{5})^2 = 0$

$$-8x^2 - 8y^2 + 40(z-\sqrt{5})^2 = 0$$