Inductive premise

$k=1$; $A = A^1$ gives you the # of walks of leutgh 1 (i.e. edges) from v_i to v_j as the entry a^i_j

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Inductive hyp.: A^{h-1} gives the # of walks of length $h-1$ from v_i to v_j as its entry b^i_j

Inductive th.: same with walks of length h and matrix A^h

Each walk of length h from v_i to v_j is the concatenation of a walk of length $h-1$ from v_i to some vertex v_m with a walk of length 1 from v_m to v_j. The # of walks of this type is $b_{im} \cdot a^m_j$. This for all vertices $m = 1, 2, \dots, \nu$

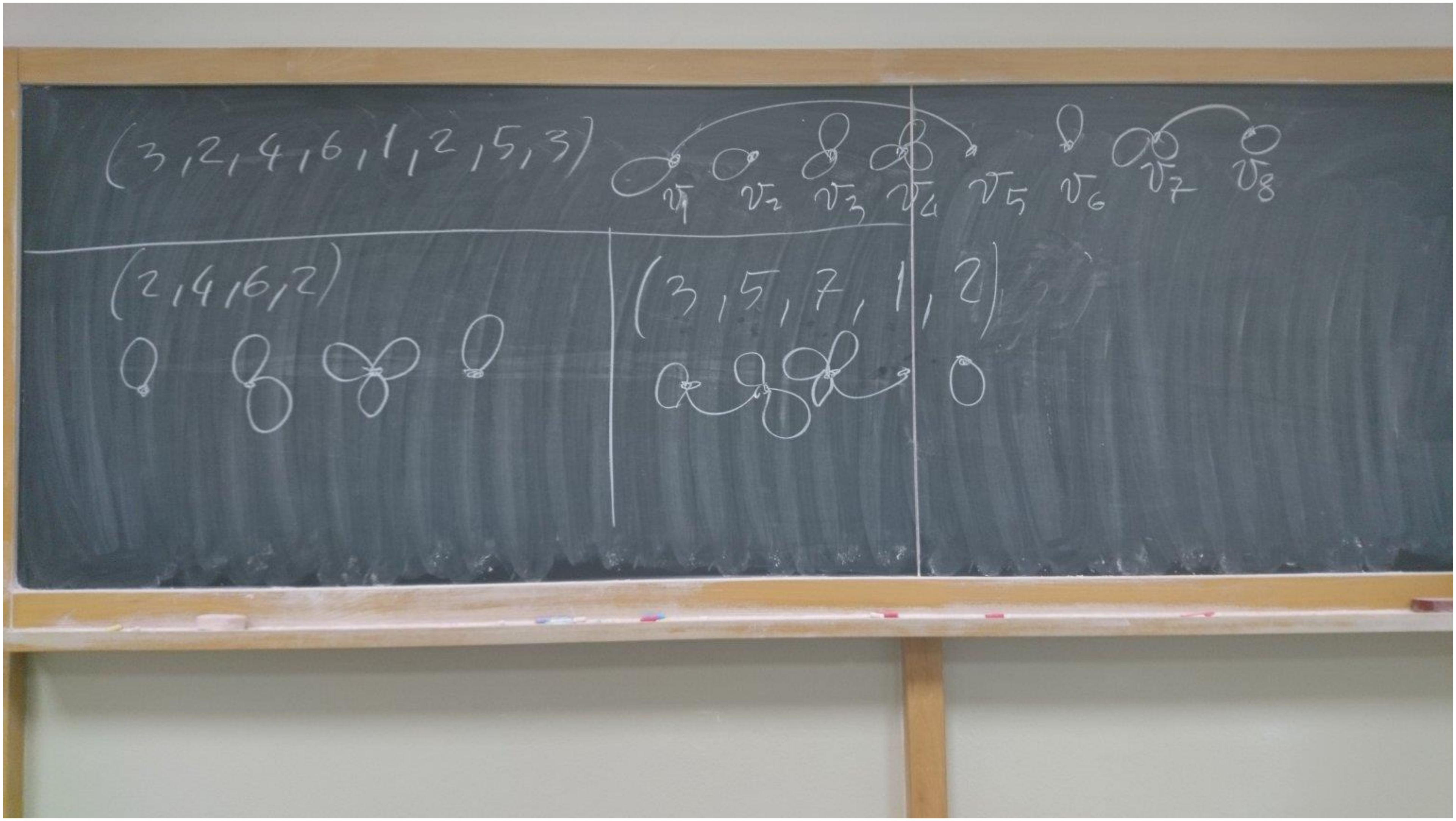
So the number of walks of length h is $\sum_{m=1}^{\nu} b^i_m a^m_j$ i.e. the (i,j)-entry of A^h

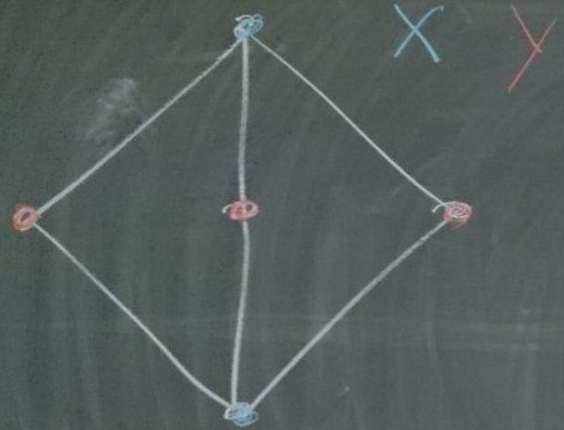
Thm G simple, $\delta \geq k$.
Th. At each vertex of G, at least one path of at least length k starts.

Inductive premise: $\delta \geq 1$. Then each vertex is the origin of at least one path of length at least 1.

Inductive hyp.: $\delta \geq k-1 \Rightarrow \exists$ path of length at least $k-1$ starting at each vertex
Inductive th.: same with k instead of $k-1$

Let $\delta(G) \geq k$.
Take any vertex v. Let edge e join v to a vertex u. Now let $G' = G - e$
$\delta(G') \geq k-1$. By inductive hypothesis, in G' a path of at least length $k-1$ starts at u; call it P. Then $e+P$ is a path of length at least k in G

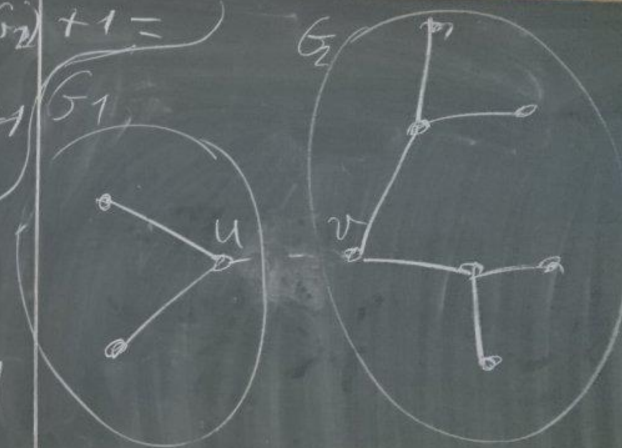

(3, 2, 4, 6, 1, 2, 5, 3)
v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8
(2, 4, 6, 2)
(3, 5, 7, 1, 2)

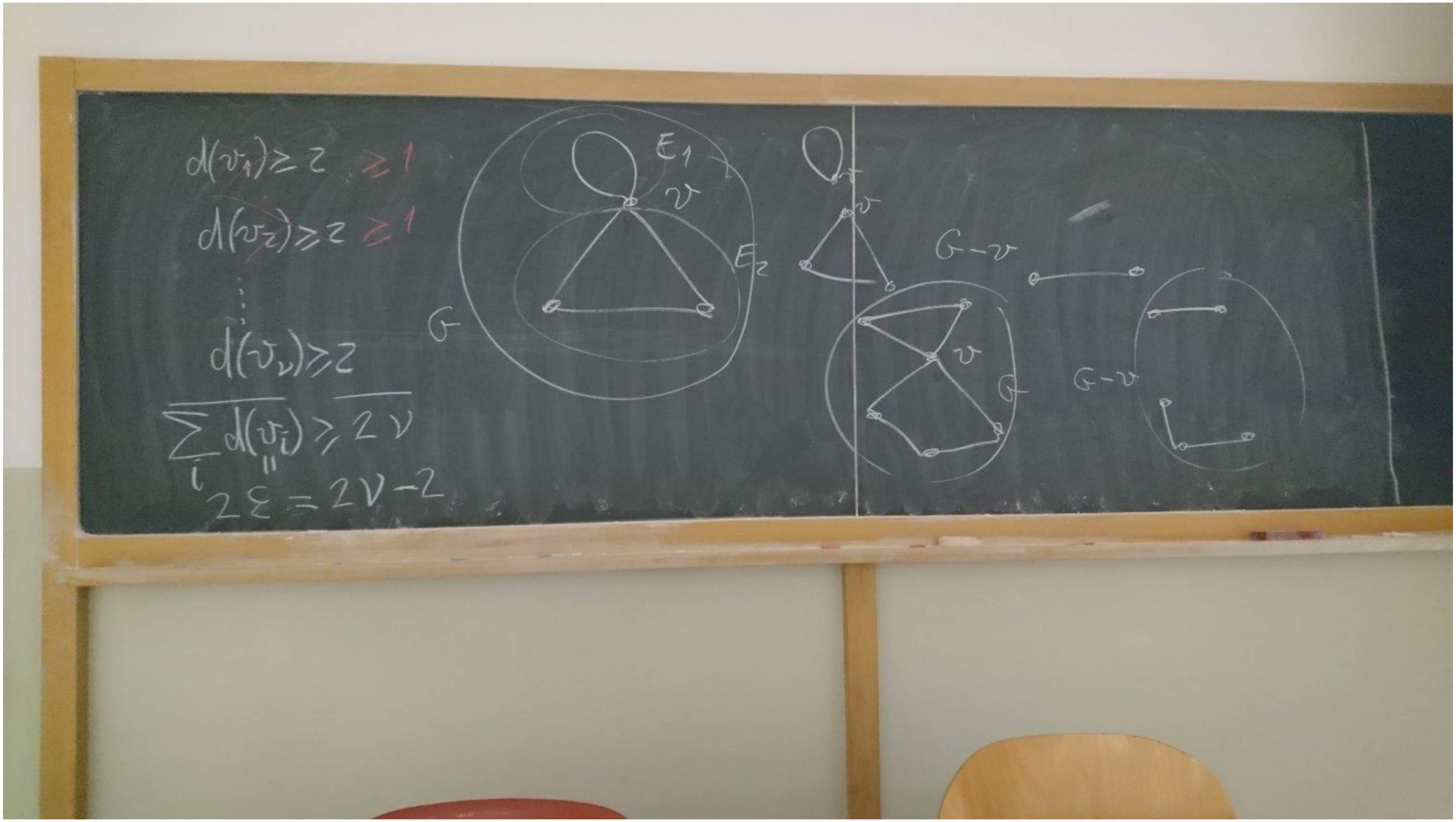


$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + 1 =$$
$$= \nu(G_1) - \cancel{1} + \nu(G_2) - 1 + \cancel{1}$$
$$= \nu(G_1) + \nu(G_2) - 1 =$$
$$= \nu(G) - 1$$

$$\varepsilon(G_1) = \nu(G_1) - 1$$

$$\varepsilon(G_2) = \nu(G_2) - 1$$





$d(v_1) \geq 2 \geq 1$
$d(v_2) \geq 2 \geq 1$
$\vdots$
$d(v_\nu) \geq 2$
$\sum_i d(v_i) \geq 2\nu$
$2\varepsilon = 2\nu - 2$
E_1
v
E_2
G
v
$G - v$
v
G
$G - v$