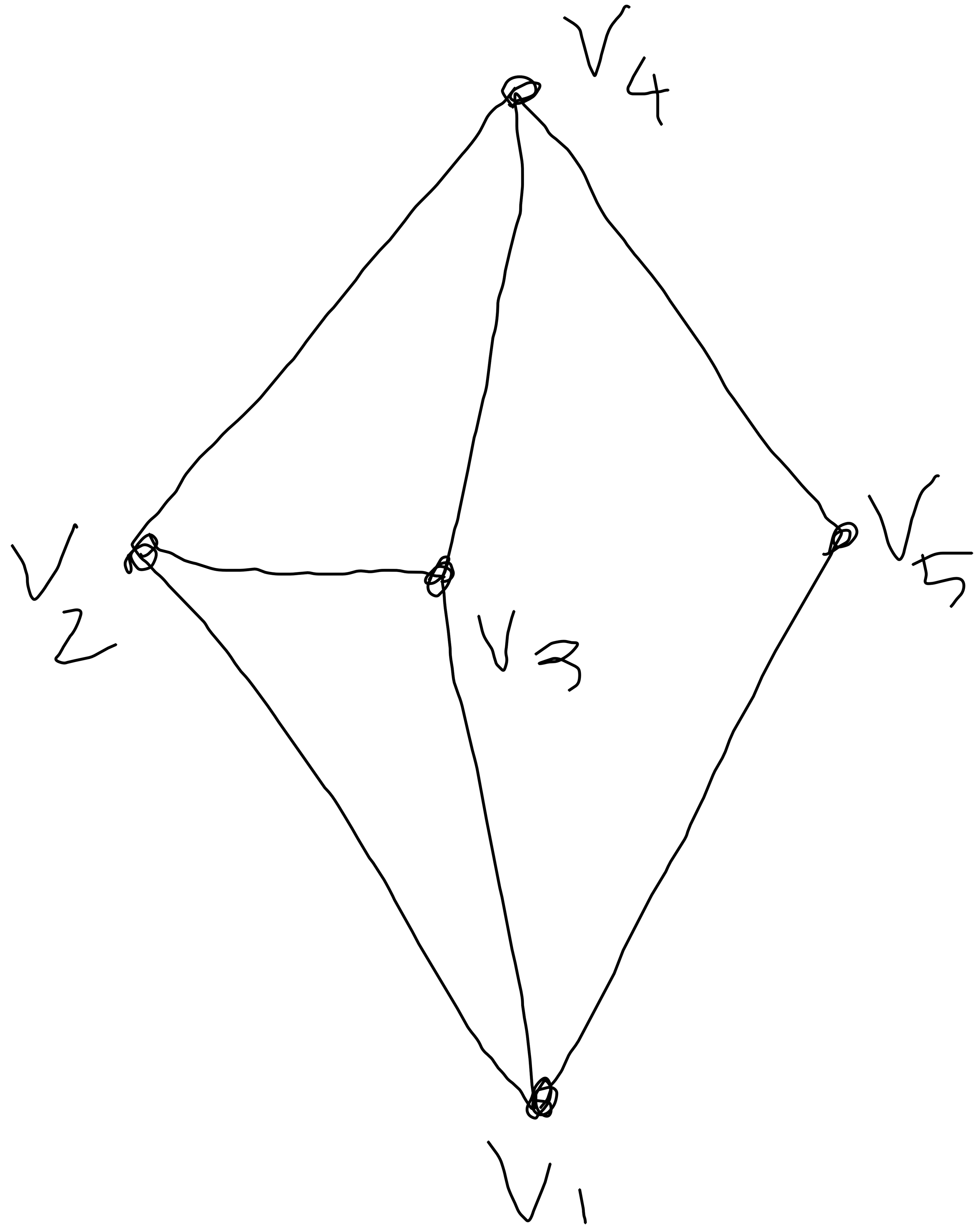


$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 1 & 1 & 3 & 0 \\ 1 & 3 & 2 & 1 & 2 \\ 1 & 2 & 3 & 1 & 2 \\ 3 & 1 & 1 & 3 & 0 \\ 0 & 2 & 2 & 0 & 2 \end{pmatrix}$$

$A^3 = ($ 1 3 … 6 5 $)$

$V_1 \to V_2$ $V_1 \to V_3$

$3 \cdot 1 + 1 \cdot 0 + 1 \cdot 0 + 3 \cdot 1 + 0 \cdot 0$

$V_1 \to V_1$ ∃ edge $V_1 - V_5$

no edge $V_2 - V_5$

no edge $V_3 - V_5$

$V_1 \to V_4$ ∃ edge $V_4 - V_5$

no 2-walks $V_1 \to V_5$

no edge $V_5 - V_5$

$(d_1\ d_2\ \cdots\ d_n)$ $\forall i$ d_i non-negative integer

$\sum_{i=1}^{n} d_i$ is even.

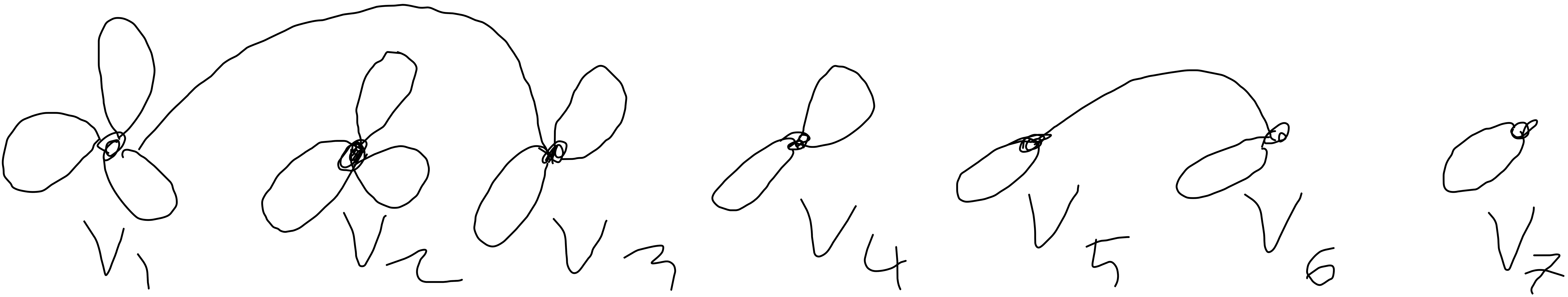
I take n vertices $V_1, \ldots, V_n$.

$\forall i$ if d_i is even I put $\frac{d_i}{2}$ loops on V_i

if d_i is odd I put $\frac{d_i-1}{2}$ loops on V_i

Finally I connect pairwise by an edge vertices V_r, V_s for which d_r, d_s are odd

$(7, 6, 5, 4, 3, 3, 2)$



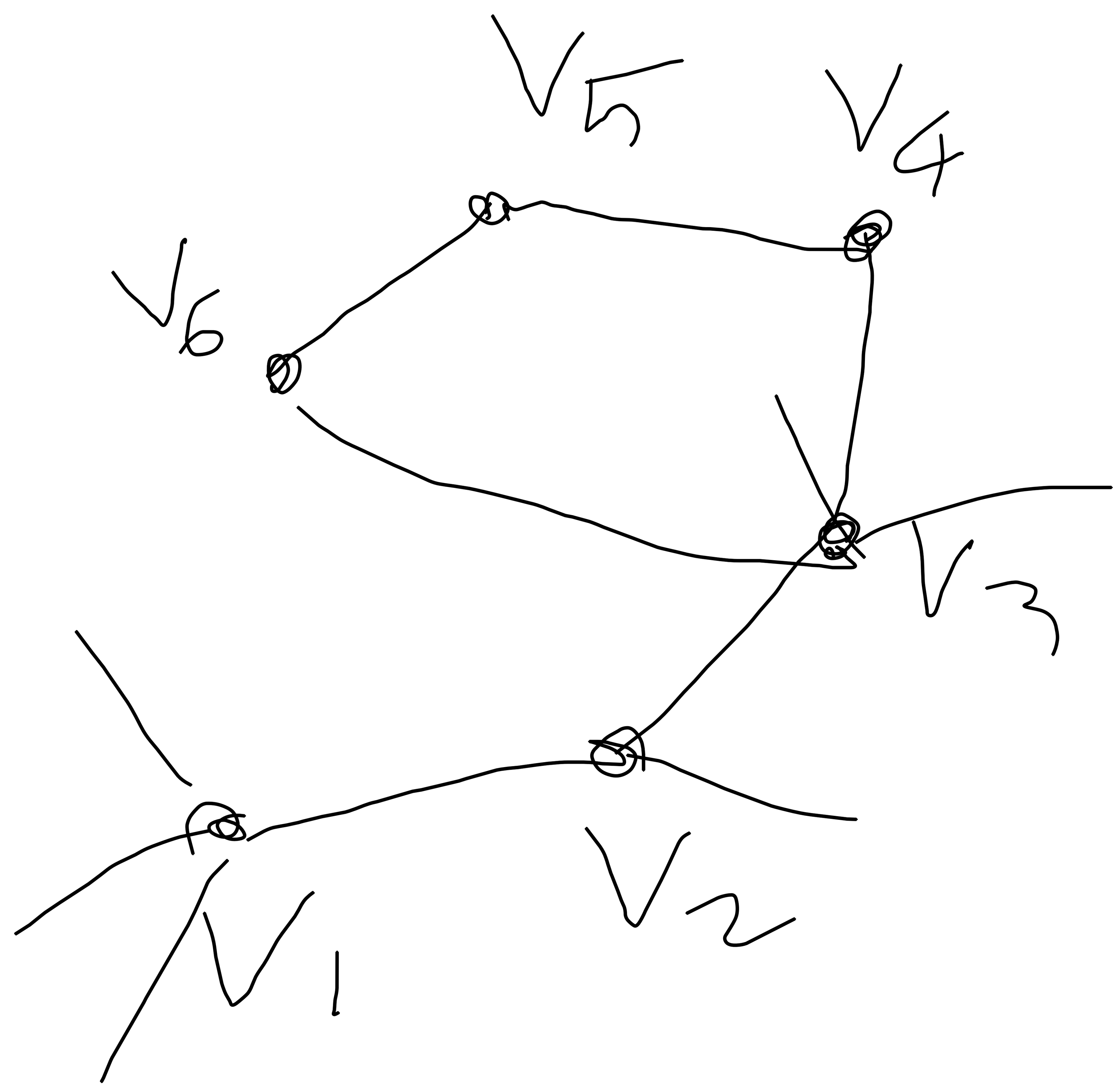
$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + 1 =$$

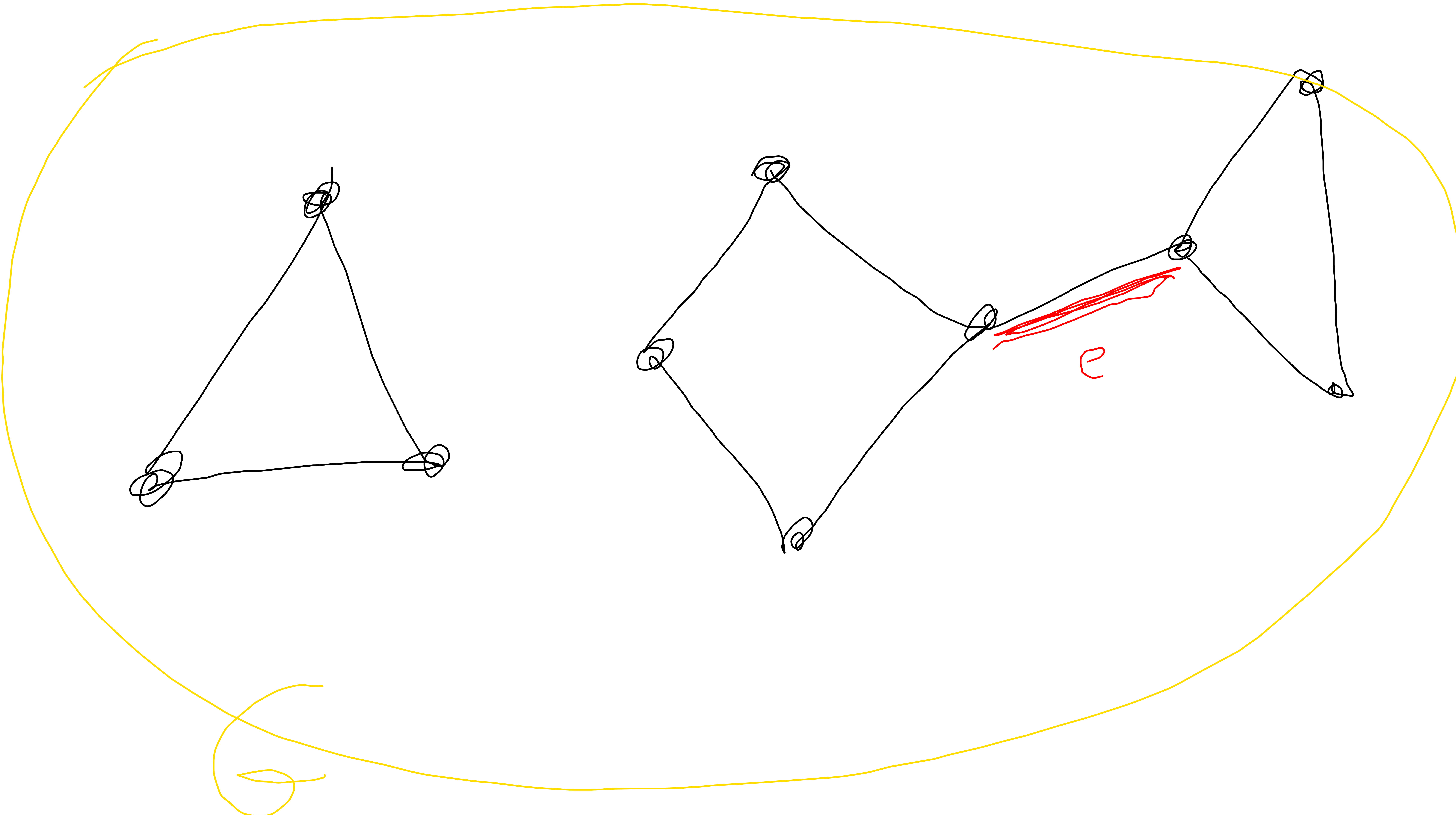
$$= \nu(G_1) \cancel{-1} + \nu(G_2) - 1 \cancel{+1} =$$

$$= \nu(G_1) + \nu(G_2) - 1 = \nu(G) - 1$$

$\delta \geq 2$

Pick up any vertex, call it v_1; use one of the edges incident on v_1 to go to another vertex, call it v_2; since $\delta \geq 2$, then $d(v_2) \geq 2$, so there is at least one edge incident on v_2 that we have not yet used. Going on like this, I always have at least one edge to exit a vertex which we have entered. But the graph is finite, so eventually we meet a vertex that we had already used. This closes a cycle.



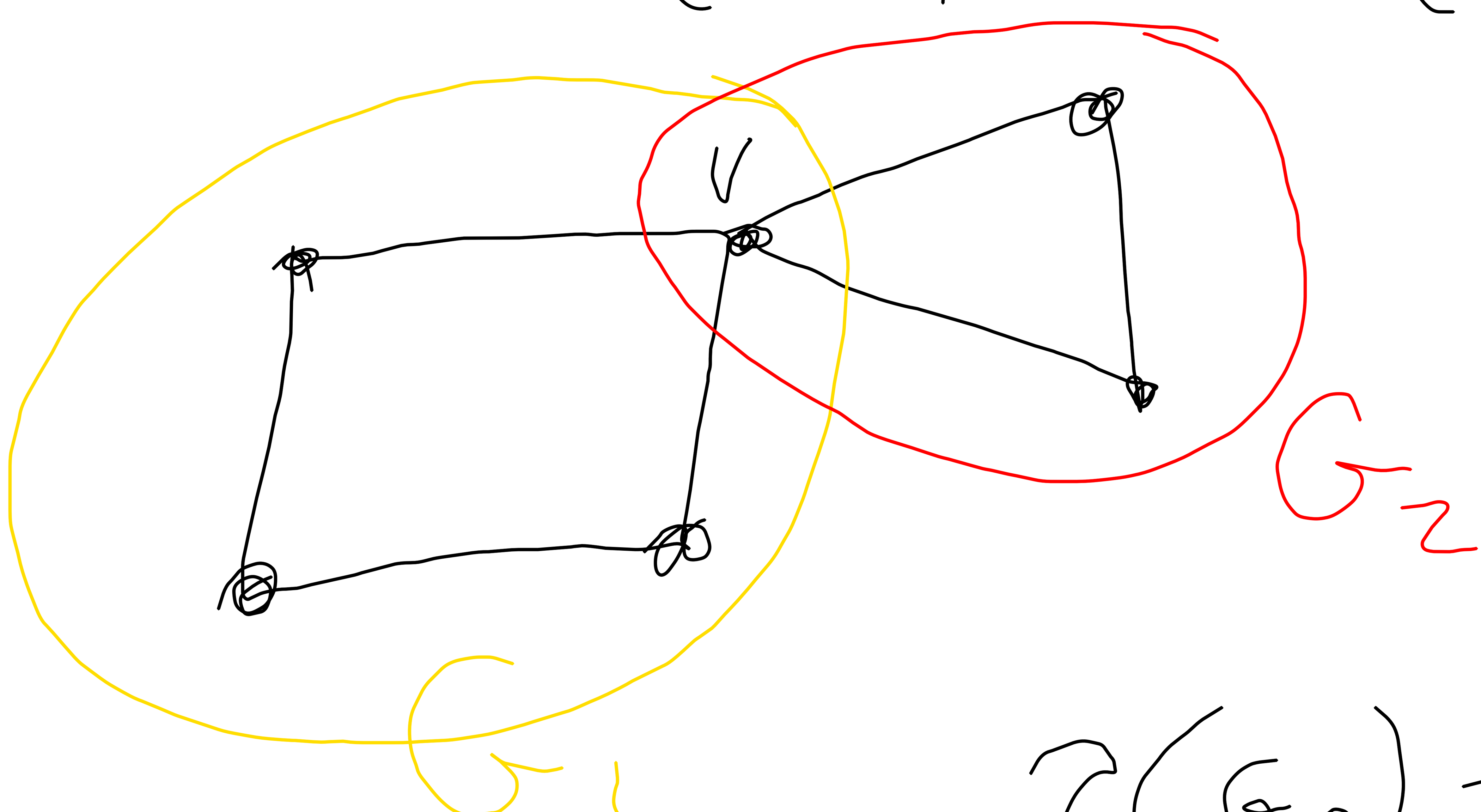

e


G
G - e
G · e

TRICKS

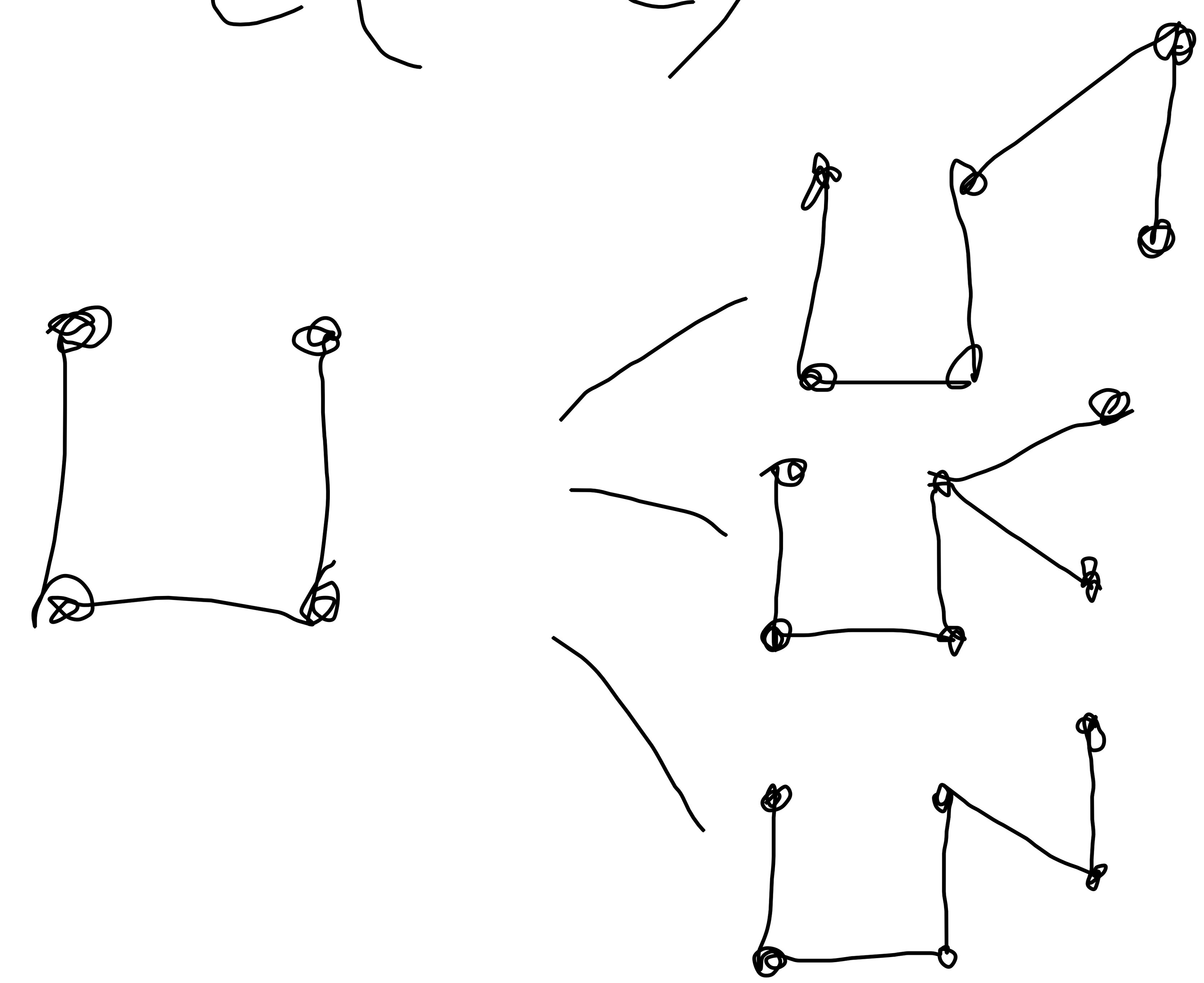
1) If G has a cut vertex v which "separates" subgraphs G_1 and G_2,

Then $\tau(G) = \tau(G_1) \cdot \tau(G_2)$

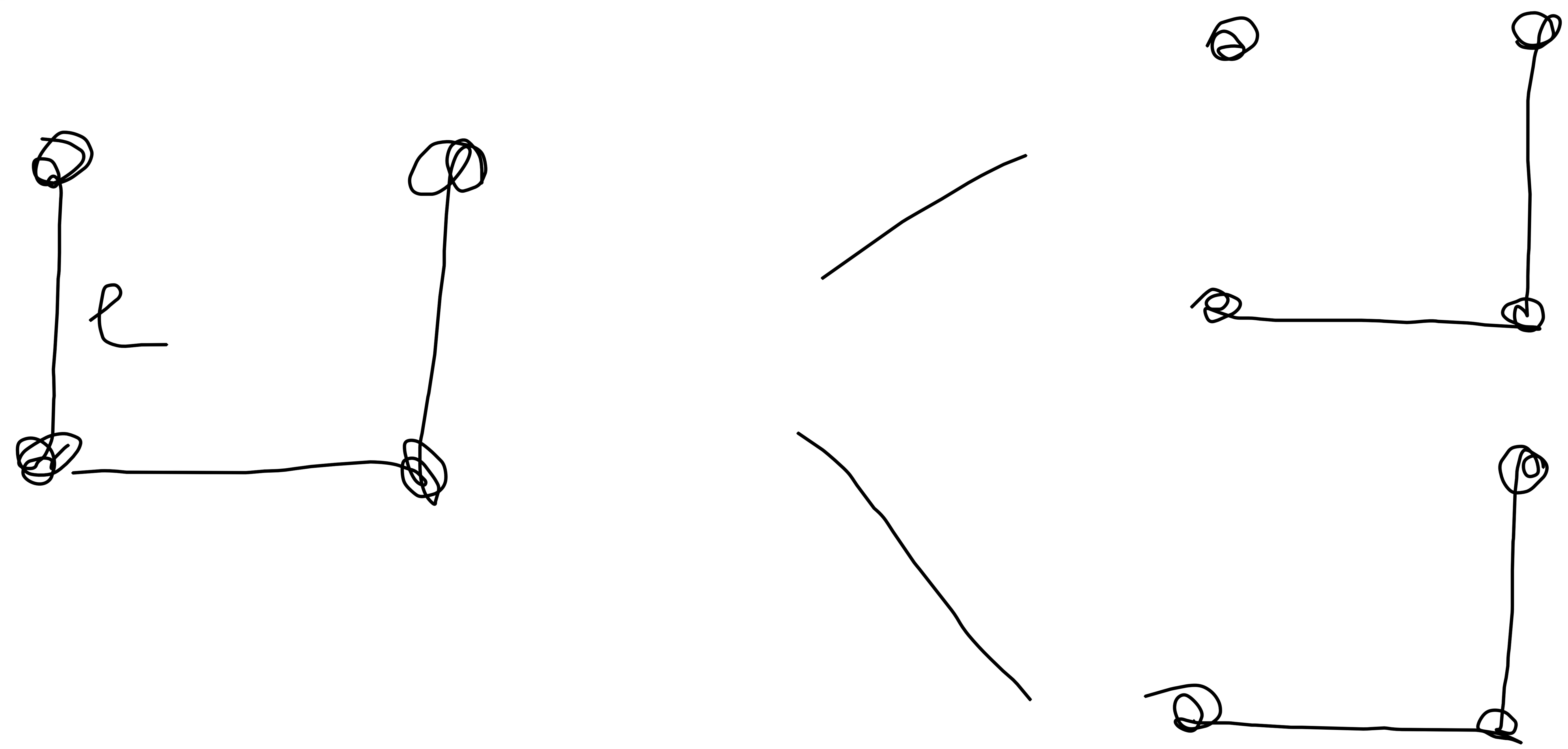


$\tau(G_2) = 3$

$\tau(G_1) = 4$



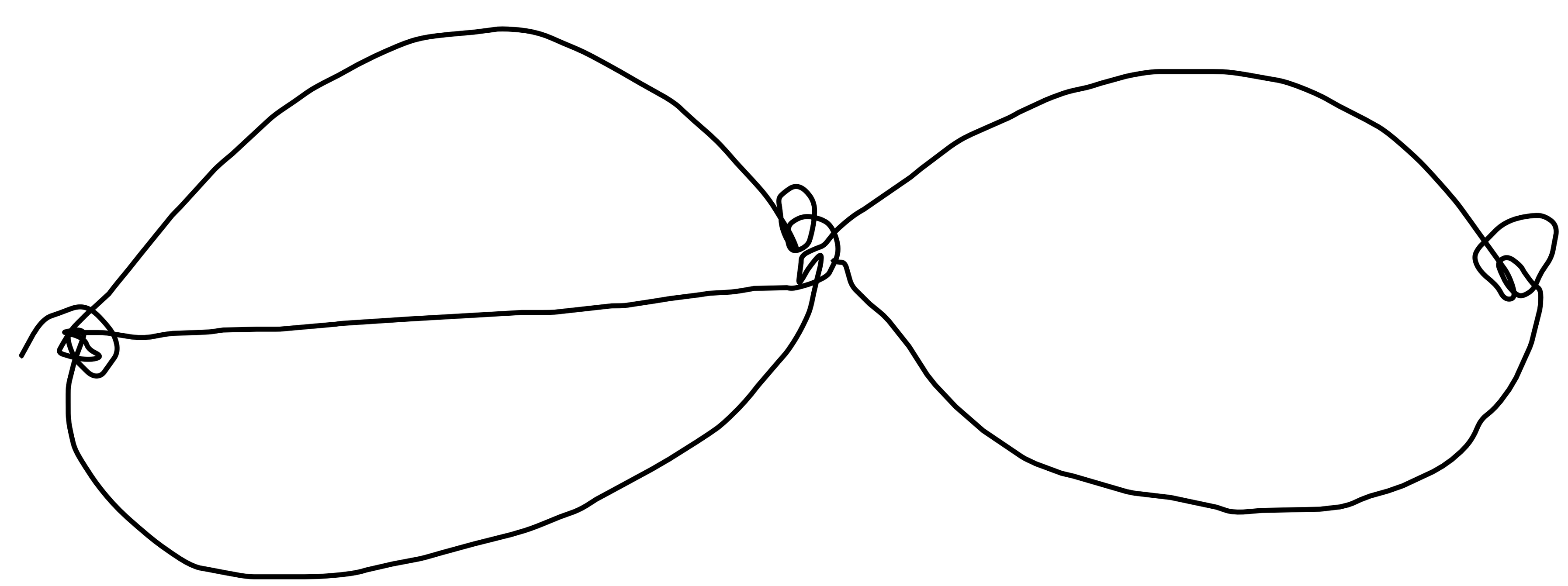
2) If G is disconnected, then $\tau(G) = 0$



3) If G is a cycle of k vertices, then $\tau(G) = k$: you get a spanning tree by deleting one and each of the edges.

4) If G is given by 2 vertices with k multiple edges between them, Then $\tau(G) = k$: each one of the multiple edges gives a spanning tree

Particular case of 1):



$3 \cdot 2 = 6$