

G a tree with exactly two vertices of degree 1.

Let u, v be the vertices of degree 1.
Necessarily all other vertices have degree 2, because

$$\sum_{v_i \in V} d(v_i) = 2E = 2V - 2$$

Start with u . There is exactly one vertex joined to it. Call it v_1 ;
apart from u , there is only one vertex joined to v_1 . Call it v_2 .

Having reached a vertex v_{h-1} , it is joined to v_h and is joined to only another vertex: call it v_h . We are forming a path $u v_1 v_2 \dots v_h$. Eventually we must reach also v (the only other vertex of degree 1).

because G is finite. No vertex has been left off, because otherwise it would belong to a different component, and this is impossible because G is a tree, so it is connected.