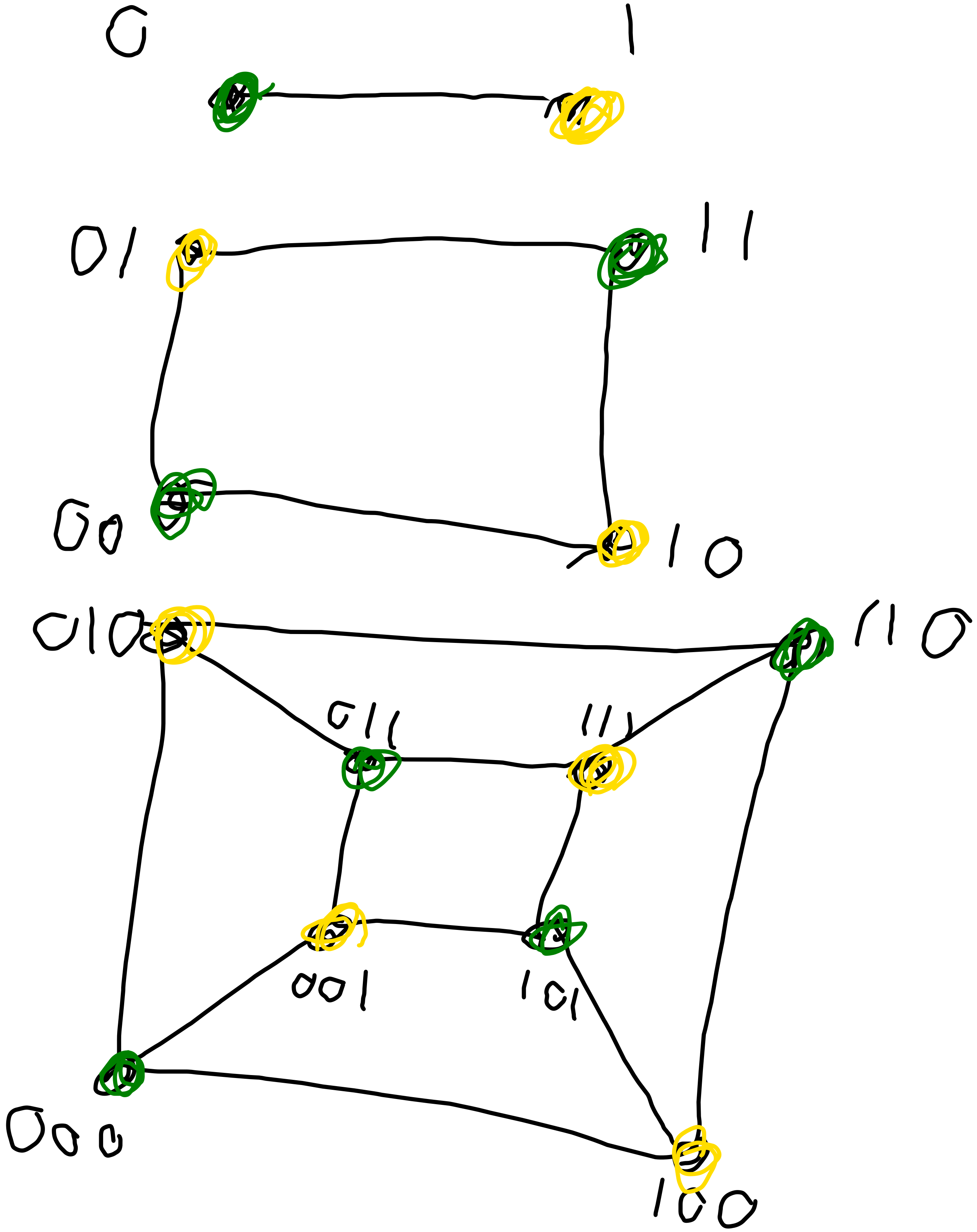

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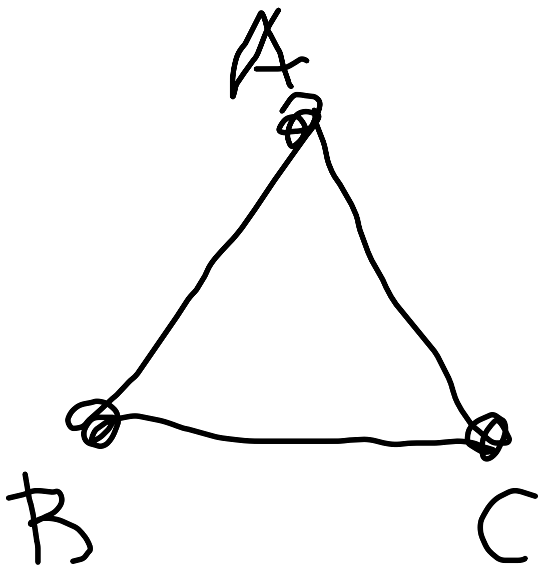
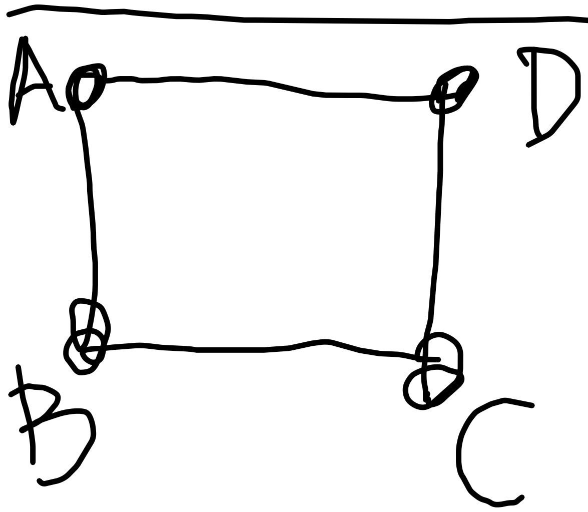




$\cancel{A \mapsto D}$
$\cancel{B \mapsto B}$
$\cancel{C \mapsto C}$
$\cancel{D \mapsto A}$

$A \mapsto A$
$B \mapsto B$
$C \mapsto C$

$A \mapsto A$
$B \mapsto C$
$C \mapsto B$

$A \mapsto B$
$B \mapsto C$
$C \mapsto A$

$A \mapsto B$
$B \mapsto A$
$C \mapsto C$

$A \mapsto C$
$B \mapsto A$
$C \mapsto B$

$A \mapsto C$
$B \mapsto B$
$C \mapsto A$

$$
\begin{array}{ll}
v_1 & \delta \le d(v_1) \le \Delta \\
v_2 & + \; \delta \le d(v_2) \le \Delta \\
\vdots & \qquad \vdots \\
v_\nu & + \; \delta \le d(v_\nu) \le \Delta \\
\hline
 & \nu \cdot \delta \le \displaystyle\sum_{i=1}^{\nu} d(v_i) \le \nu \cdot \Delta
\end{array}
$$

$$\sum_{i=1}^{\nu} d(v_i) = 2\varepsilon$$

$$\nu\delta \le 2\varepsilon \le \nu\Delta$$

$$\frac{1}{\nu}$$

$$\delta \le \frac{2\varepsilon}{\nu} \le \Delta$$

Def - Two sets A, B have the same cardinality if there exists at least one bijection from A to B.

$$|A| = |B| \overset{\text{def}}{\iff} \exists f : A \to B$$

f bijective

Def - The cardinality of a set C is said to be greater than or equal to the one of set D if there is at least one injective map from $C \to D$

$|C| \leq |D| \overset{def}{\iff} \exists\, g : C \to D$
g injective

PROP – $|C| \leq |D| \iff \exists\, h : D \to C$
h surjective

$A = |\{\text{Students of UniBo Eng.}\}| \sim 10.000 > |\{\text{days in a year}\}| = B = 366$

this implies that $\nexists$ injective map from A to B

$$\Big|\underbrace{\{\text{people}\}}_{C=}\Big| \sim 7{,}000{,}000{,}000 > \Big|\underbrace{\{\text{possible numbers of hairs on a body}\}}_{D=}\Big|$$

$$1{,}000{,}000$$

$\nexists$ injective map from C to D

$$E = \{\text{people in the group}\} \qquad F = \{\text{set of possible degrees}\} =$$

$$| \quad | = 2 \quad > \quad = \underbrace{\{1, 2, \dots, 19\}}_{=19}$$

$\nexists$ injective map from E to F

(2, 5, 3, 4, 7, 6, 1)

Alternative and equivalent definitions

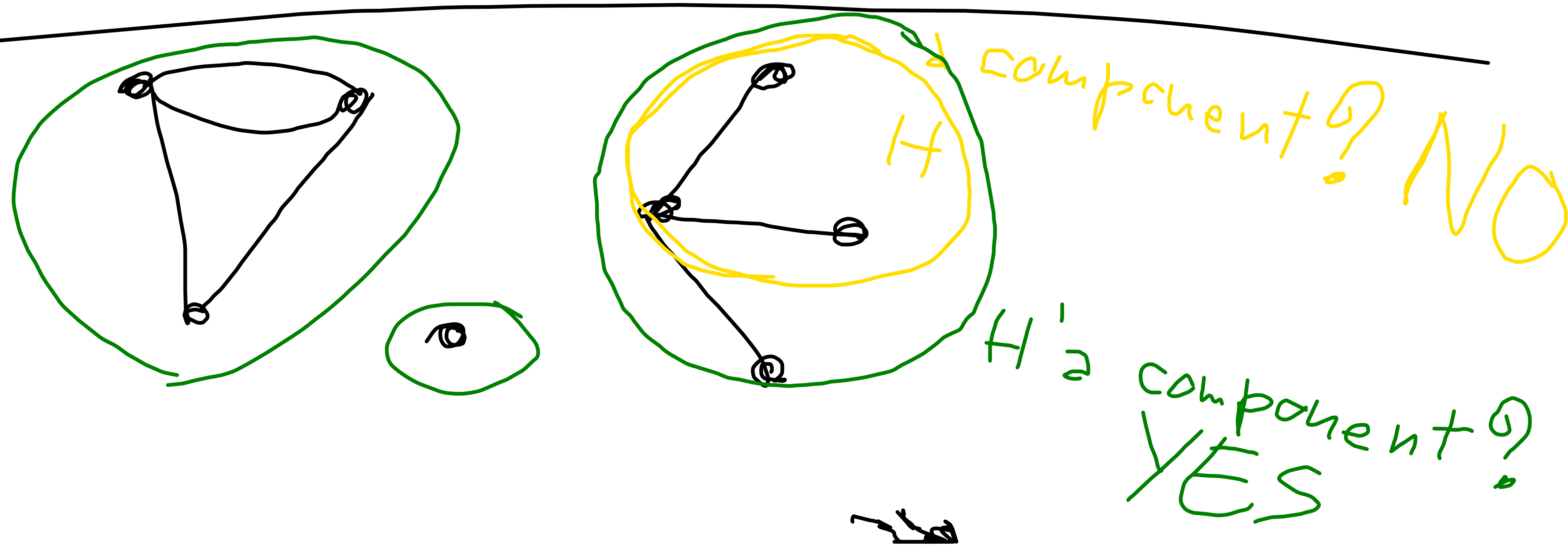
A graph G is said to be connected if for any two vertices u, v of G there is at least one (u, v)-path.

A component of a graph G is a maximal connected subgraph of G

A maximal subgraph of G with a certain property is

1) a subgraph H of G with the given property

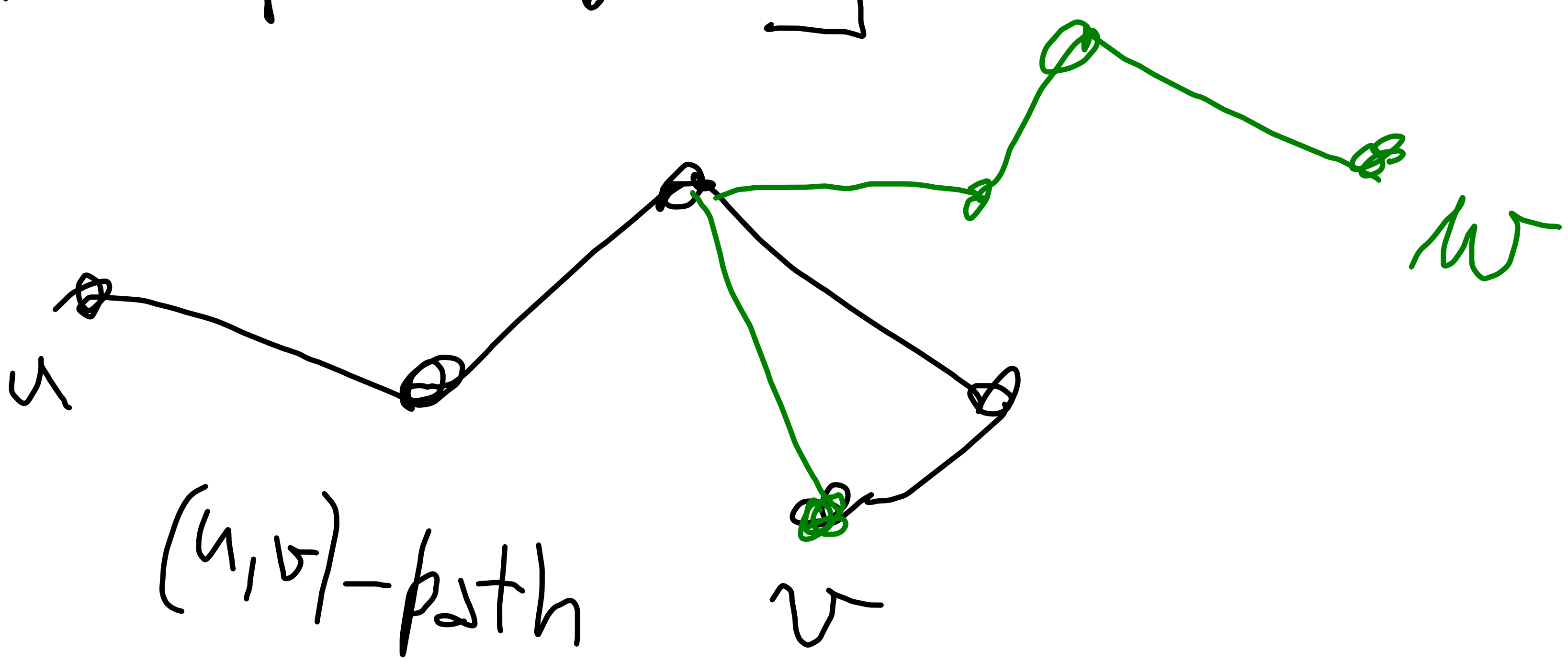
2) such that no subgraph H' of G properly containing H has that property.



$\exists$ (u,v)-path $\quad u = v_0 e_1 v_1 e_2 \cdots e_k v_k = v$

$\exists$ (v,w)-path $\quad v = w_0 f_1 w_1 f_2 \cdots f_h w_h = w$

$\exists$ (u,w)-path? $\quad \exists\ u = v_0 e_1 v_1 \cdots e_k v_k = v = w_0 f_1 w_1 \cdots f_h w_h = w$



A distance on a set X is a map

$d: X \times X \longrightarrow \mathbb{R}$ such that $\forall x, y, z \in X$

1) $d(x,y) \geq 0$ $= 0 \iff x = y$

2) $d(x,y) = d(y,x)$

3) $d(x,y) + d(y,z) \geq d(x,z)$

path h path k

walk of length $h+k$ from x to z

$\longrightarrow$ path of length $\leq h+k$ from x to z