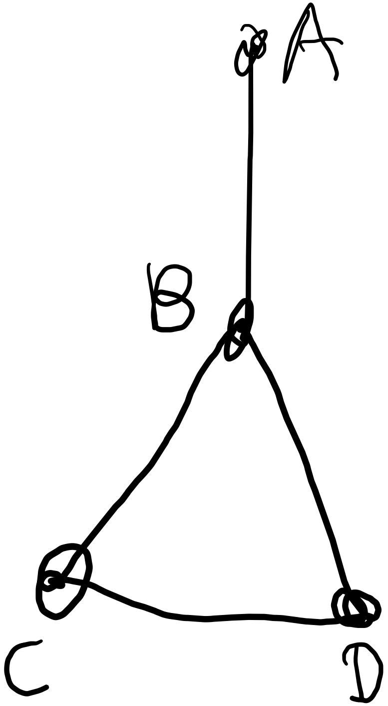
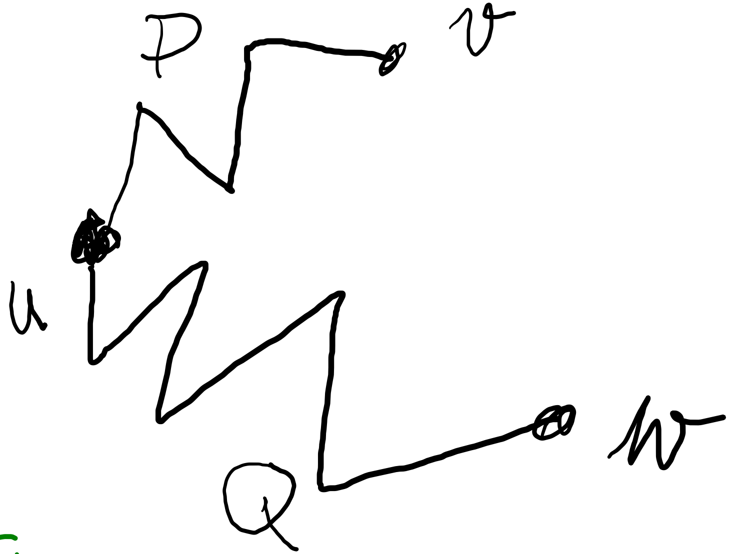
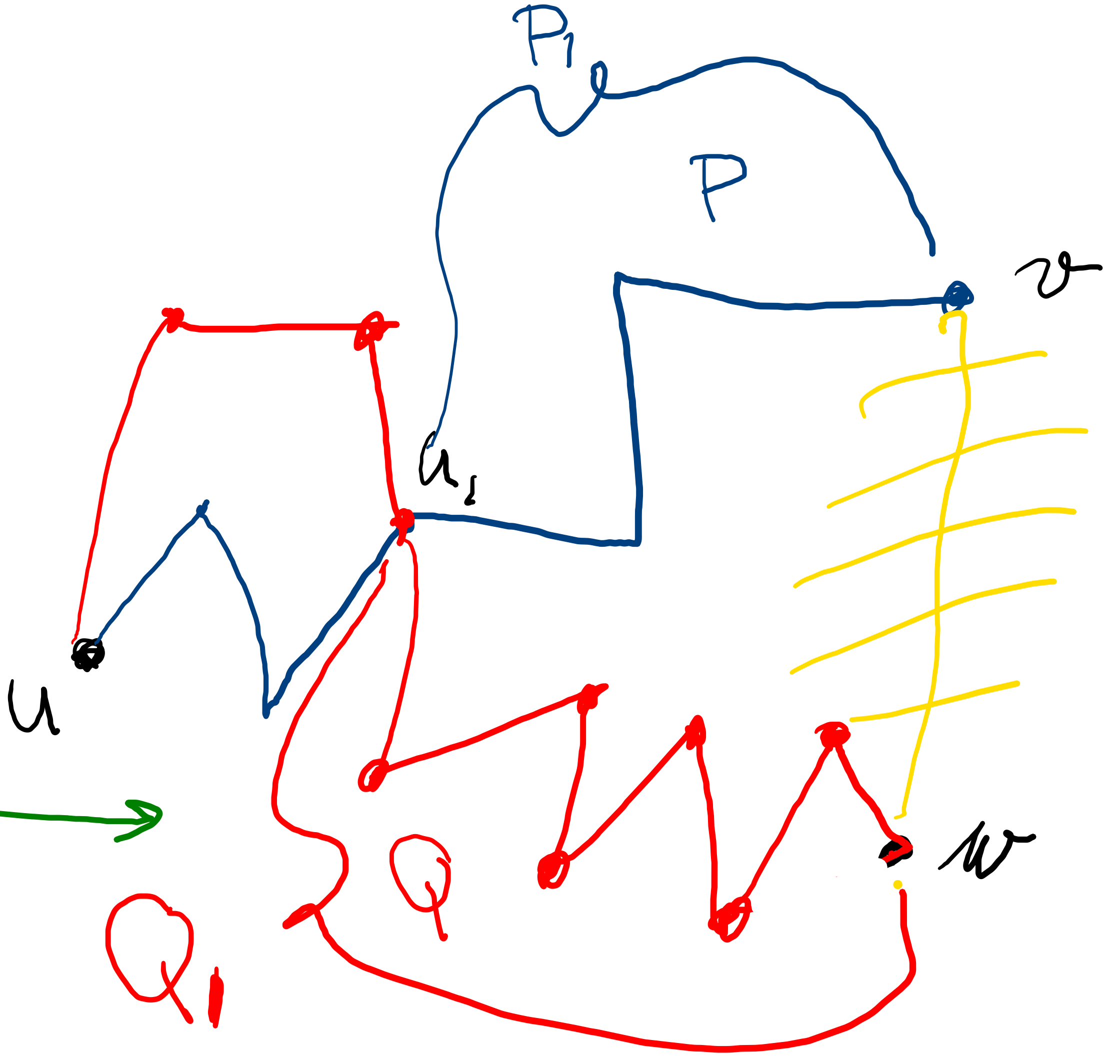


$$
\begin{aligned}
A &\mapsto A\\
B &\mapsto B\\
C &\mapsto C\\
D &\mapsto D
\end{aligned}
\qquad\qquad
\begin{aligned}
A &\mapsto A\\
B &\mapsto B\\
C &\mapsto D\\
D &\mapsto C
\end{aligned}
$$



The one drawn during class was wrong (Q had an odd number of edges)



Proof by induction

You want to prove a statement depending on a positive integer n

---

Inductive premise:

prove the statement for $n=1$

Inductive step: from the

Inductive hypothesis: the statement is true for $k-1$

prove the

Inductive thesis: the statement is true for k

Example: call $P(n)$ the number of permutations of n objects.

Theorem: $P(n) = n!$

Possible orderings of those objects.

E.g.

1	2	3
2	3	1
3	1	2
1	3	2
2	1	3
3	2	1

$P(3) = 6$

$6 = 3 \cdot 2 \cdot 1 = 3!$

Inductive premise: what is $P(1)$

$P(1) = 1 = 1!$ ✓

Inductive step:

Inductive hyp.: $P(k-1) = (k-1)!$

I want to prove that $P(k) = k!$

Let's list all $P(k-1)$ permutations of the numbers $1, \dots, (k-1)$

$P(k-1)$

1 2 3 .... $(k-2)$ $(k-1)$

3 2 1 .....

$(k-3)$ 5 2 ...

⋮

k 1 2 3 ... $(k-2)(k-1)$

1 k 2 3 ... $(k-2)$ $(k-1)$

1 2 k 3 ... $(k-2)$ $(k-1)$

1 2 3 ... $(k-2)$ k $(k-1)$

1 2 3 ... $(k-2)(k-1)$ k

$\Big\}\, k$

k 3 2 1 ....

3 k 2 1 ...

3 2 k 1

......

Surely $P(k) = k \cdot P(k-1) \underset{\text{out of Ind. Hyp}}{=} k \cdot (k-1)! = k!$

1 2 3
2 3 1
3 1 2
1 3 2
2 1 3
3 2 1

4 1 2 3
1 4 2 3
1 2 4 3
1 2 3 4

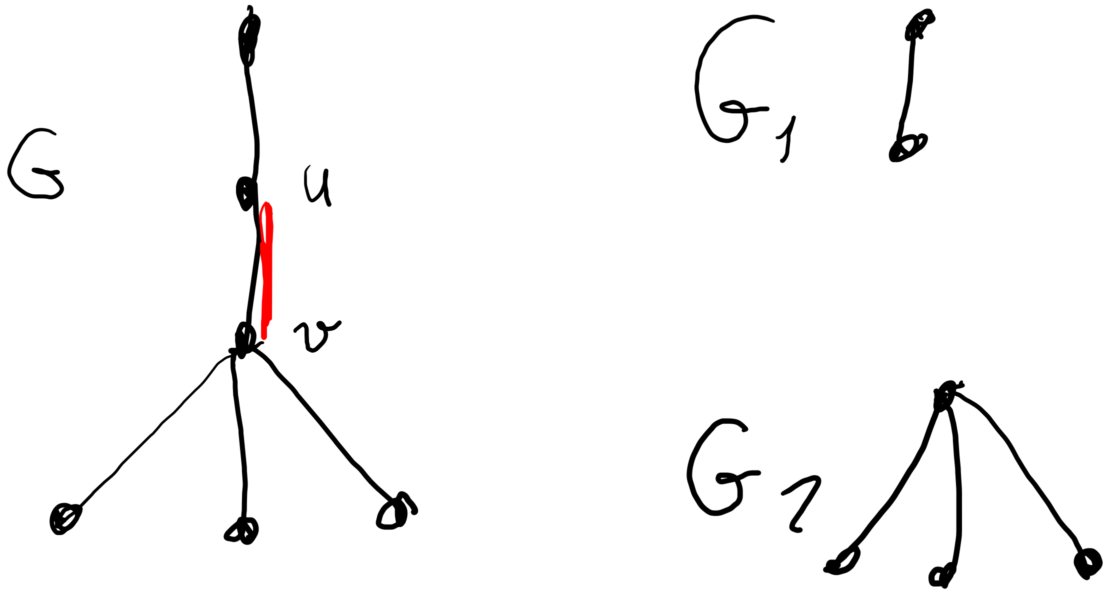
4 2 3 1
2 4 3 1
2 3 4 1
2 3 1 4

4 3 1 2
3 4 1 2
3 1 4 2
3 1 2 4

$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + \underbrace{1}_{uv} =$$

$$= \nu(G_1) \cancel{-1} + \nu(G_1) - 1 \cancel{+1} =$$

$$= \nu(G) - 1$$


G
u
v
G_1
G_1