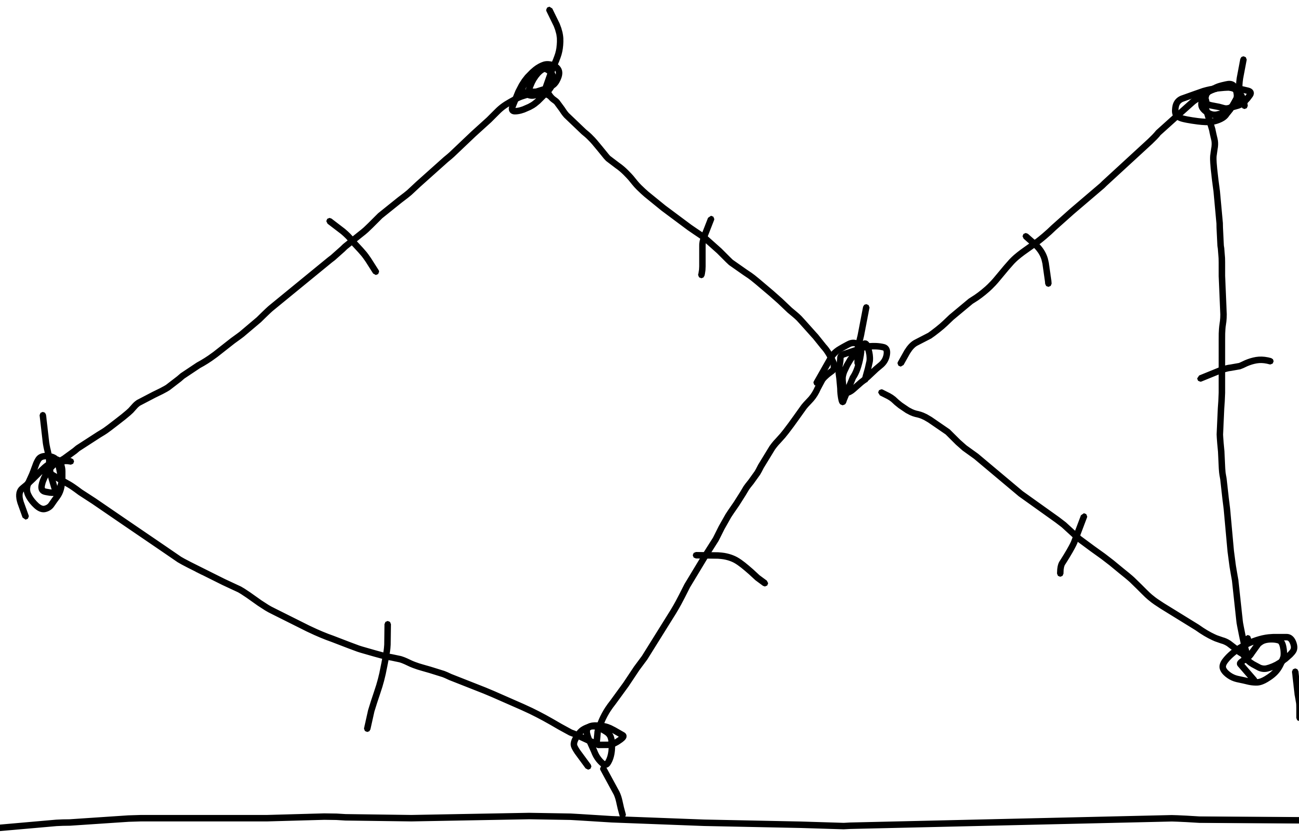
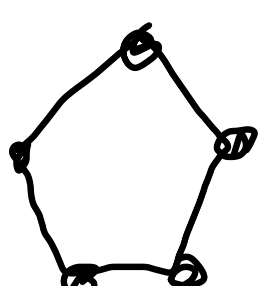


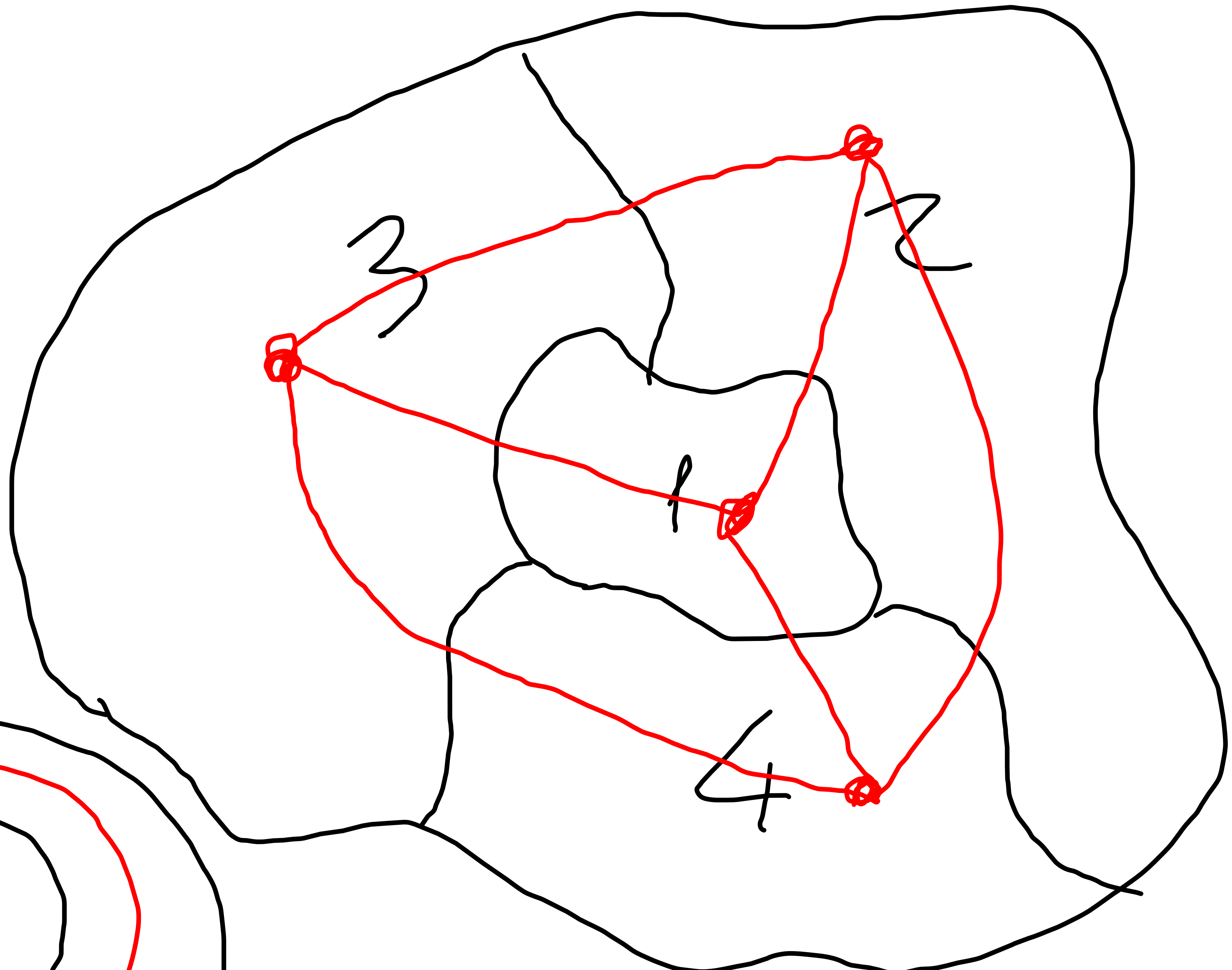
does $r(3,3)$ exist?

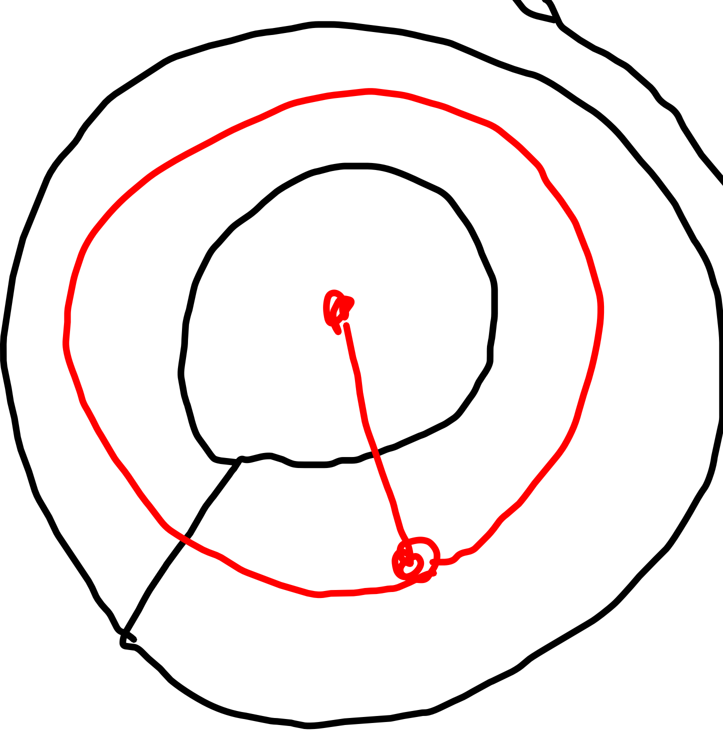
We know the existence of $r(2,3)=3$ and $r(3,2)=3$

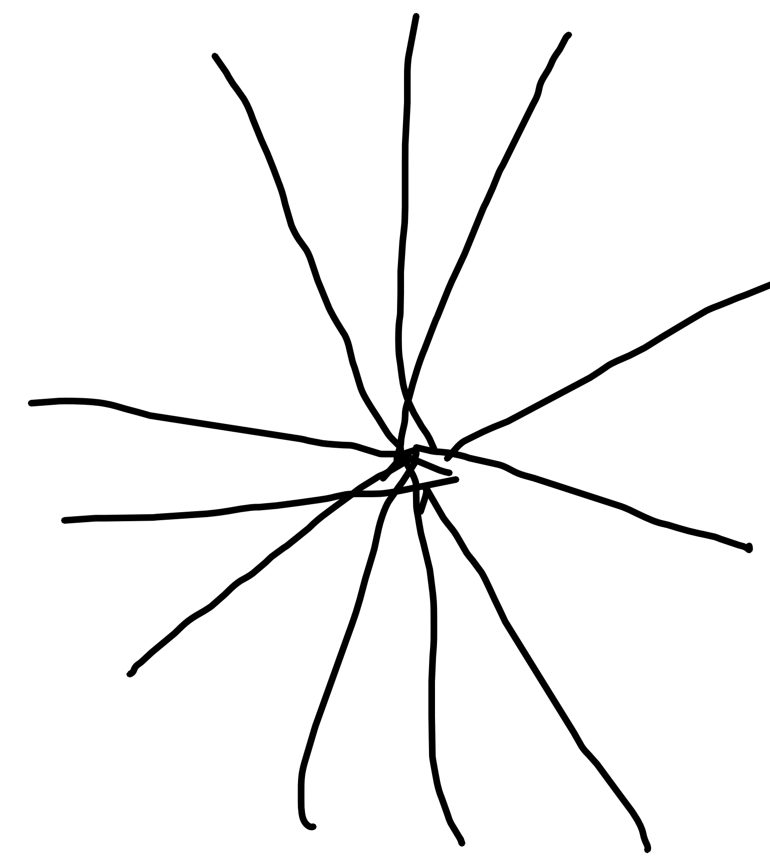
$r(3,3) \le r(2,3) + r(3,2) = 6$

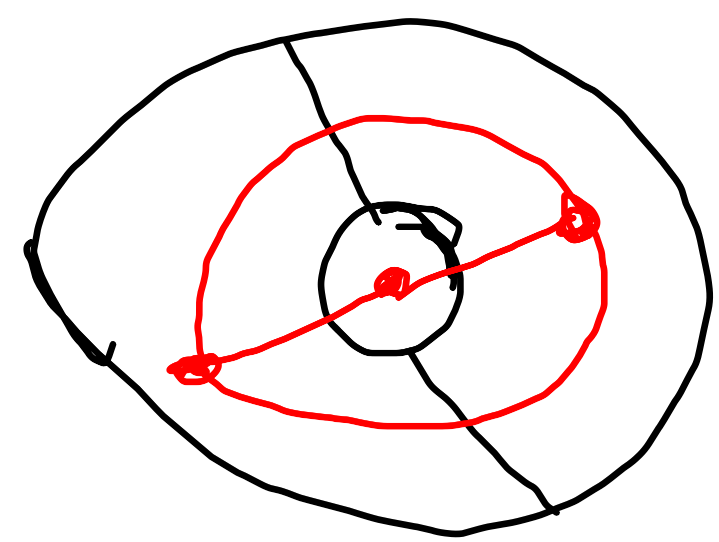


$\Rightarrow r(3,3)=6$


3
2
1
4





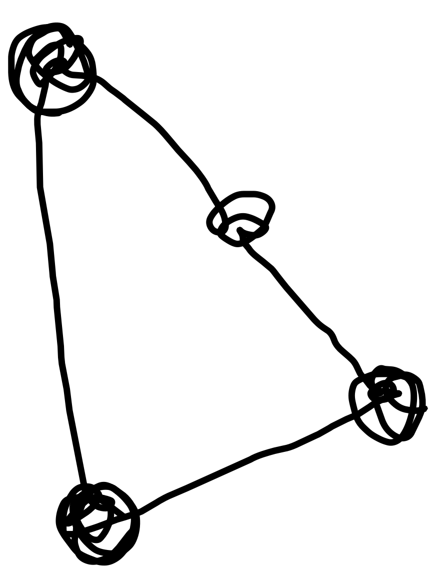


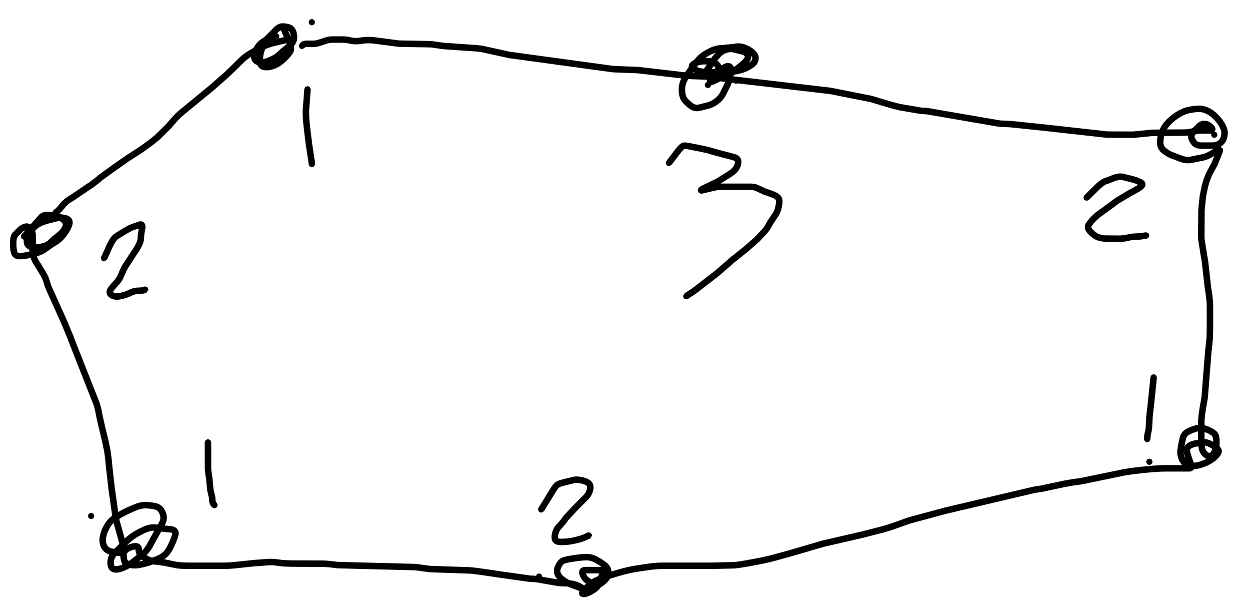
$\chi(G) = k$

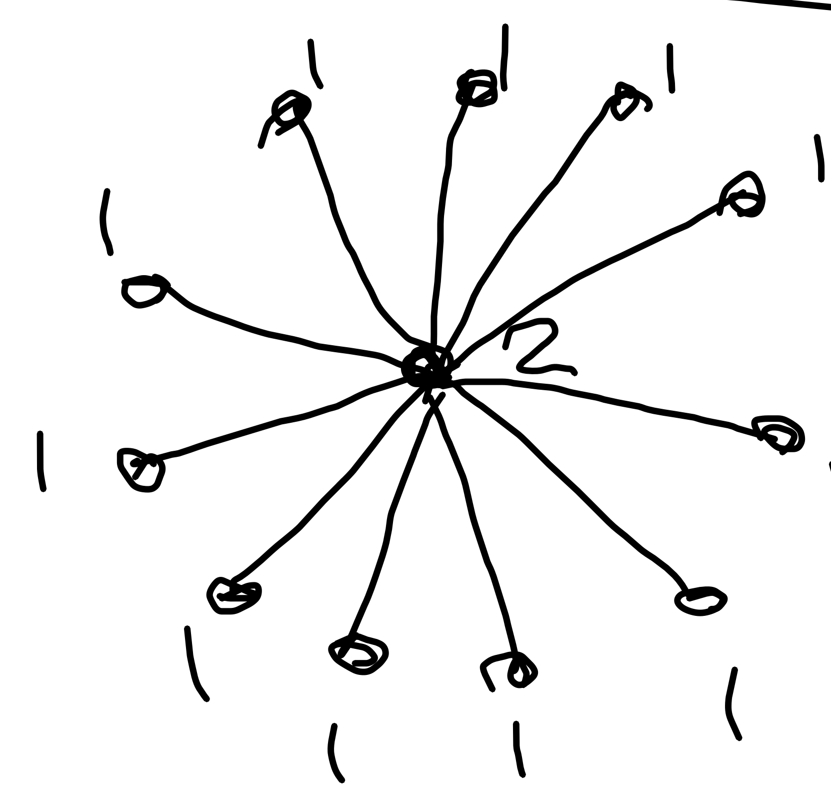
Then $\exists\ v \in V(G)$ such that

$d(v) \geq k-1$

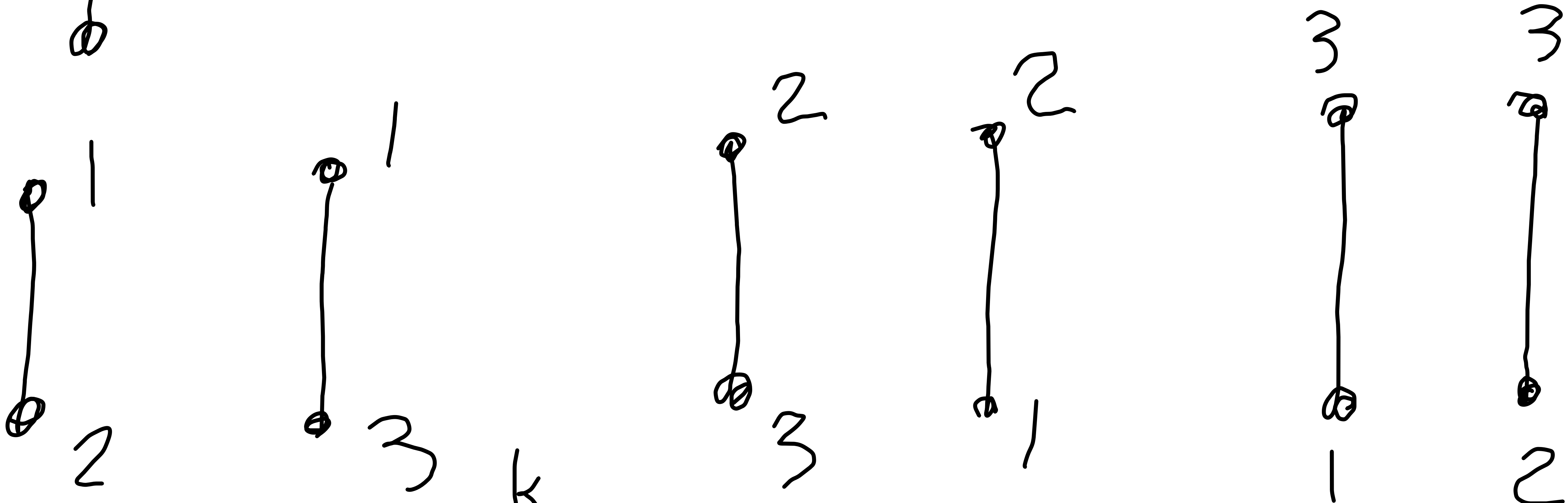
$k \leq d(v)+1 \leq \Delta+1$



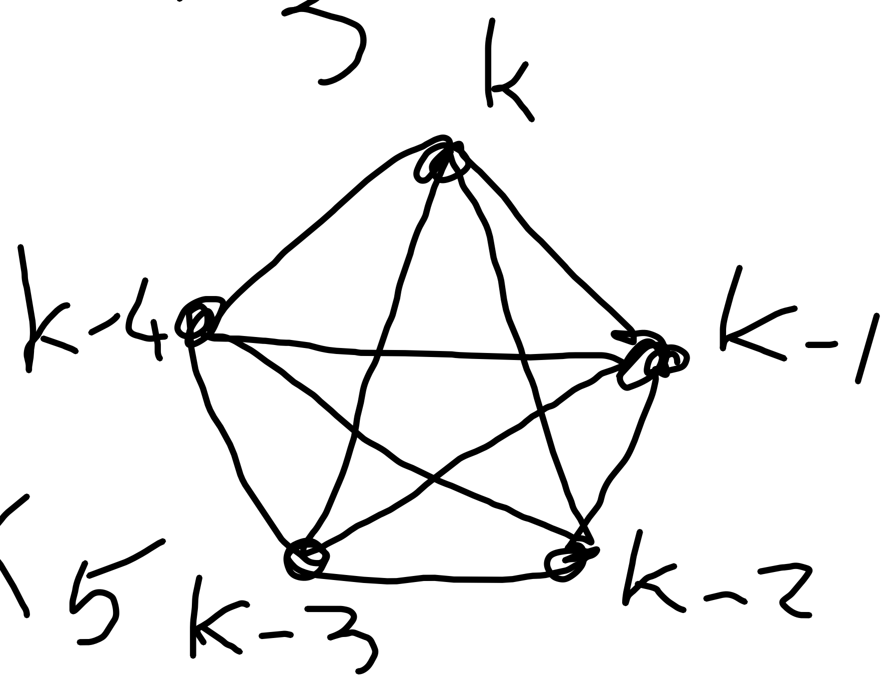




G $\quad \pi_3(G) = 6$

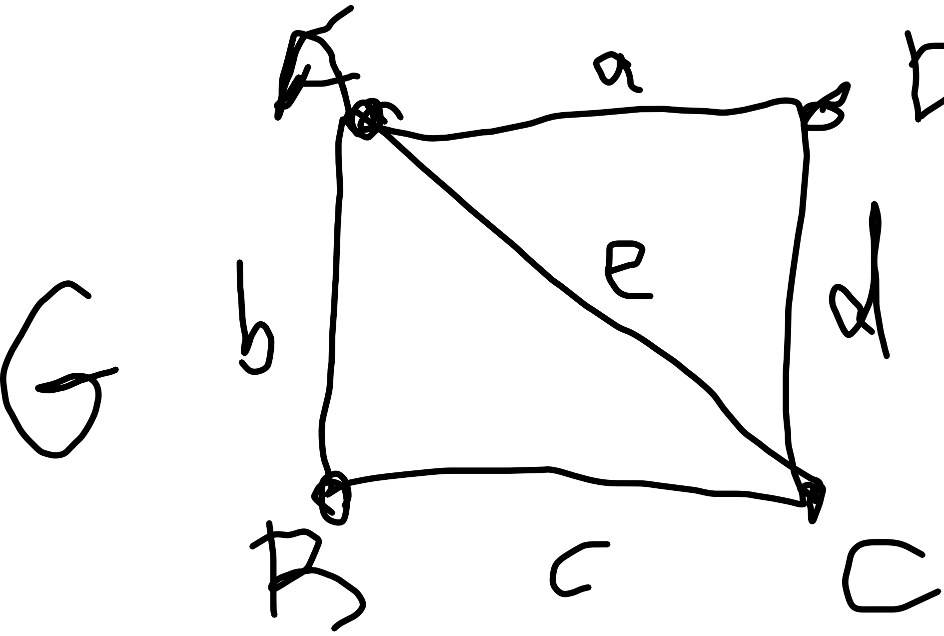


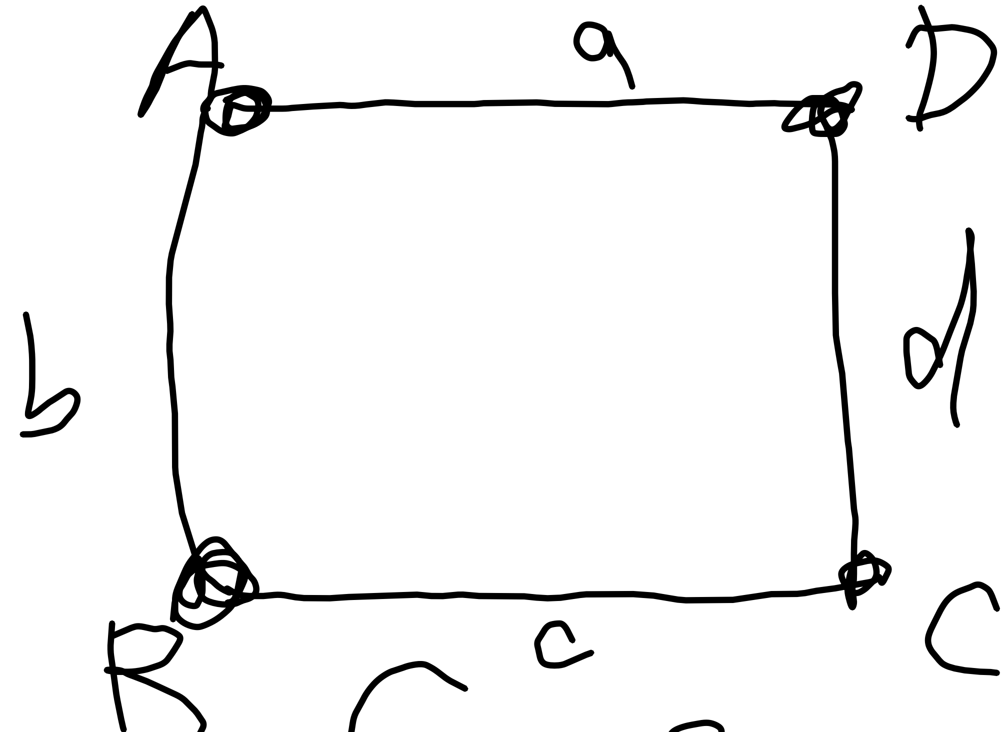
$k-5+1 =$



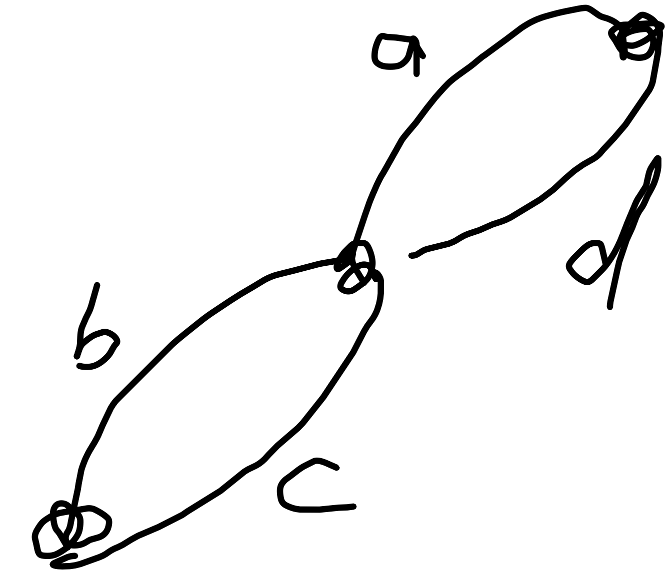
K_5

$\pi_k(K_5) = k(k-1)(k-2)(k-3)(k-4)$



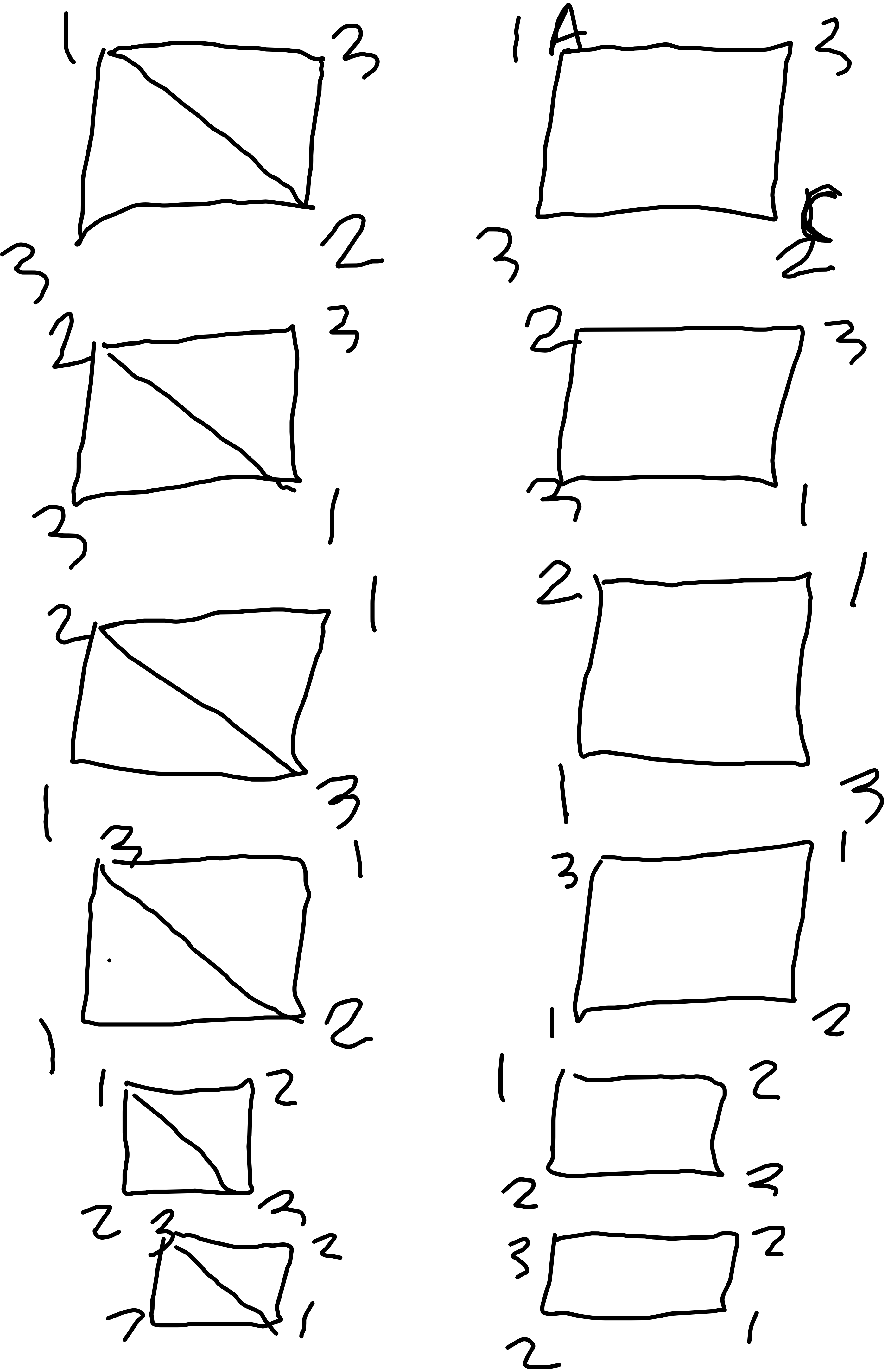


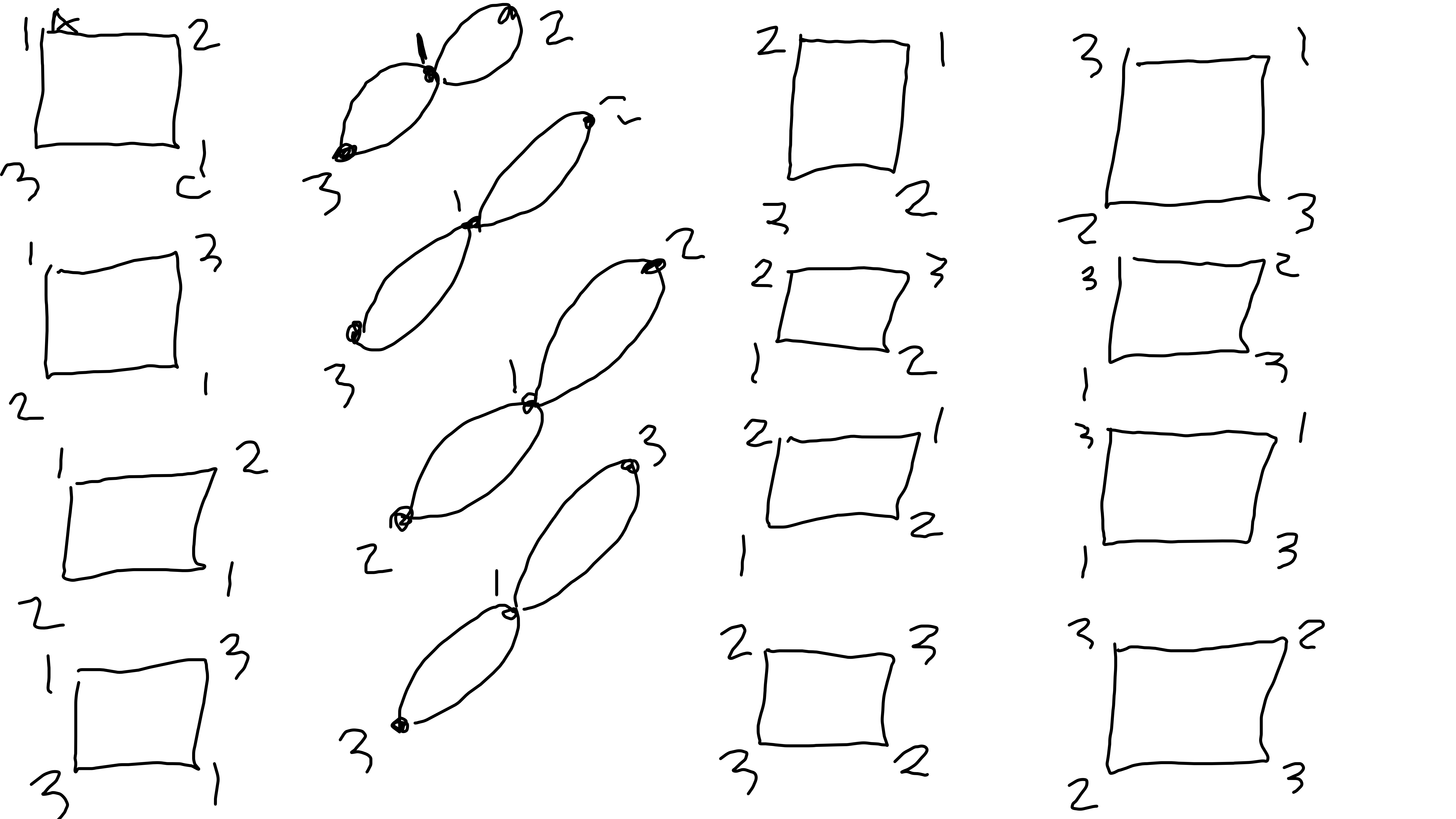
G - e



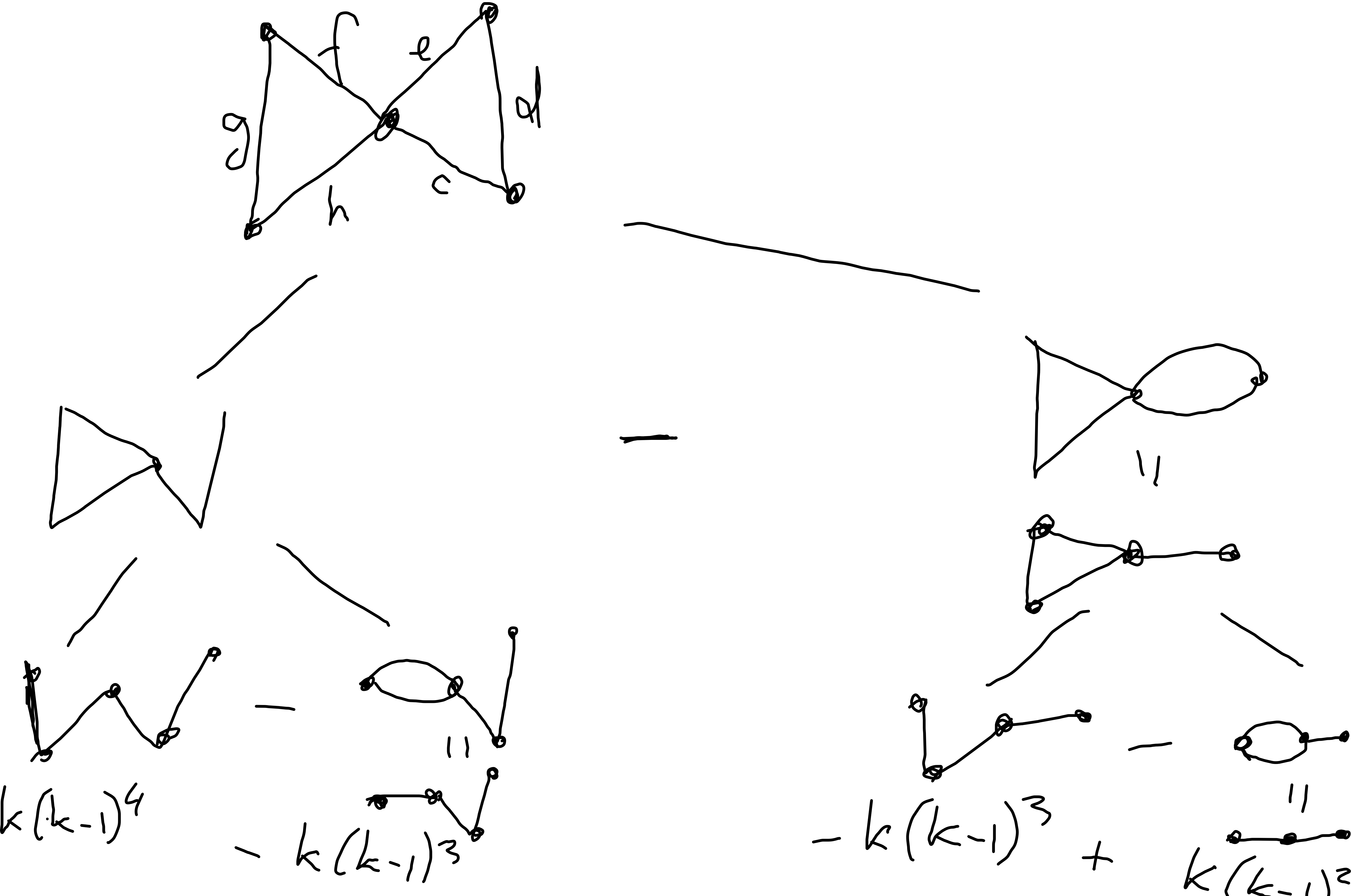
G · e

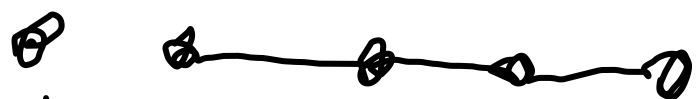
$k = 3$





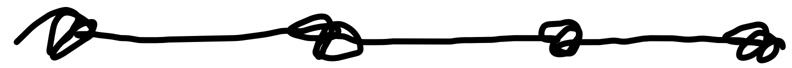
$$\Pi_3(G-e) = \Pi_3(G) + \Pi_3(G\cdot e) \Leftrightarrow \Pi_3(G) = \Pi_3(G-e) - \Pi_3(G\cdot e)$$


f
e
g
d
c
h
−
=
−
=
k(k−1)^4
− k(k−1)^3
=
−
=
− k(k−1)^3
+
k(k−1)^2



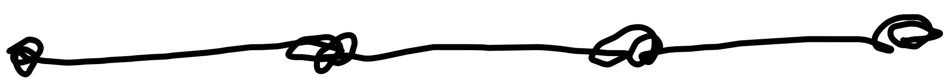
$k \qquad k(k-1)^3$

$-$



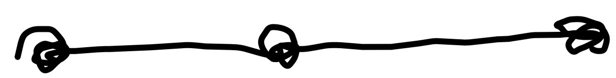
$k(k-1)^3$

$$= k(k-1)^3(k-1) =$$
$$= k(k-1)^4$$



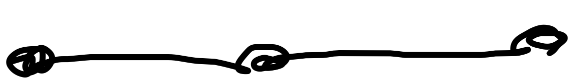
$k \qquad k(k-1)^2$

$-$



$k(k-1)^2$

$$= k(k-1)^2(k-1) =$$
$$= k(k-1)^3$$



$$= k(k-1)(k-1) = k(k-1)^2$$

$k \qquad k(k-1)$

$k^2(k-1)$

$=$

$k(k-1)$

Power $n = 1$	0 1 1 0	$x + y \quad (x+y)^n$
$n = 2$	0 1 2 1 0	$x^2 + 2xy + y^2$
$n = 3$	1 3 3 1	$x^3 + 3x^2y + 3xy^2 + y^3$
$n = 4$	1 4 6 4 1	$x^4 + 4x^3y + 6x^2y^2 + 3xy^3 + y^4$

$k(k-1)^4$

$$k(k-1)^4 - 2k(k-1)^3 + k(k-1)^2 = k(k^4-4k^3+6k^2-4k+1) + -2k(k^3-3k^2+3k-1) + k(k^2-2k+1)$$

$$\begin{array}{r} k^5 - 4k^4 + 6k^3 - 4k^2 + k \quad + \\ -2k^4 + 6k^3 - 6k^2 + 2k \\ + k^3 - 2k^2 + k \\ \hline k^5 - 6k^4 + 13k^3 - 12k^2 + 4k \end{array}$$

$+ \quad - \quad + \quad - \quad +$

a constant term of 0

alternating ok!

n. of vertices ok!

-n. of edges ok!

1 6 13 12 4 ok!