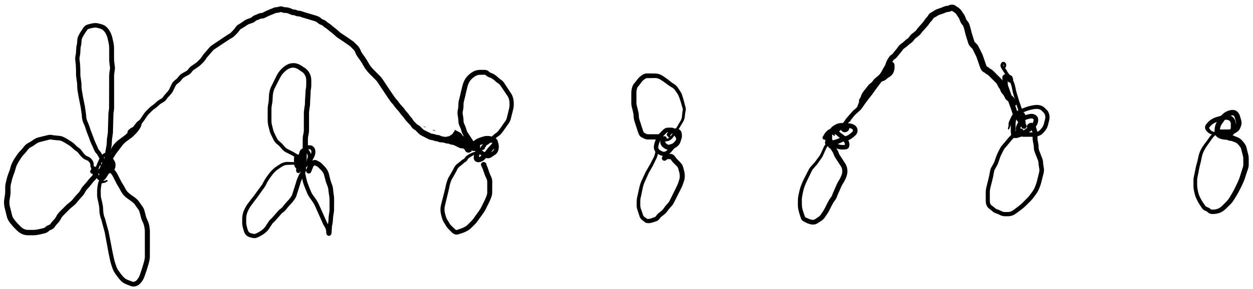
$(d_1, d_2, \ldots, d_n)$ $\quad \sum_{i=1}^{n} d_i$ even

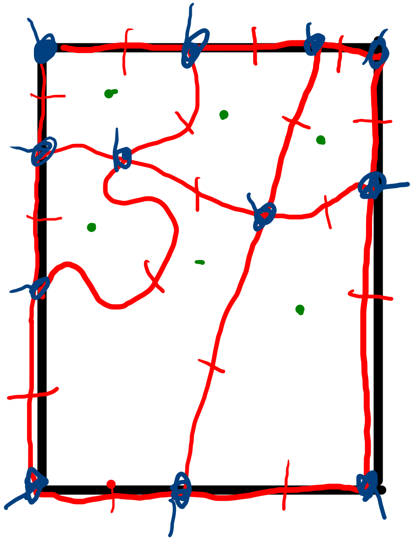
Put $\frac{d_i}{2}$ loops on the vertices with d_i even

$\frac{(d_i-1)}{2}$ on the vertices with d_i odd

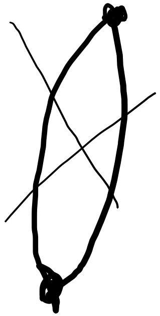
We are left with an even number of vertices which should have an odd degree

$(7, 6, 5, 4, 3, 3, 2)$





$12 - 17 + 6 = 1$



$$
\begin{array}{rl}
3 & \leq d(f_1) \\
+\ 3 & \leq +\, d(f_2) \\
& \vdots \\
+\ 3 & \leq +\, d(f_\phi) \\
\hline
3\phi & \leq \sum_{i=1}^{\phi} d(f_i) = 2\varepsilon
\end{array}
\implies \phi \leq \frac{2}{3}\varepsilon
$$

$$2 = v - \varepsilon + \phi \leq v - \varepsilon + \frac{2}{3}\varepsilon = v - \frac{\varepsilon}{3}$$

$$2 \leq v - \frac{\varepsilon}{3} \qquad \frac{\varepsilon}{3} \leq v - 2 \qquad \varepsilon \leq 3v - 6$$

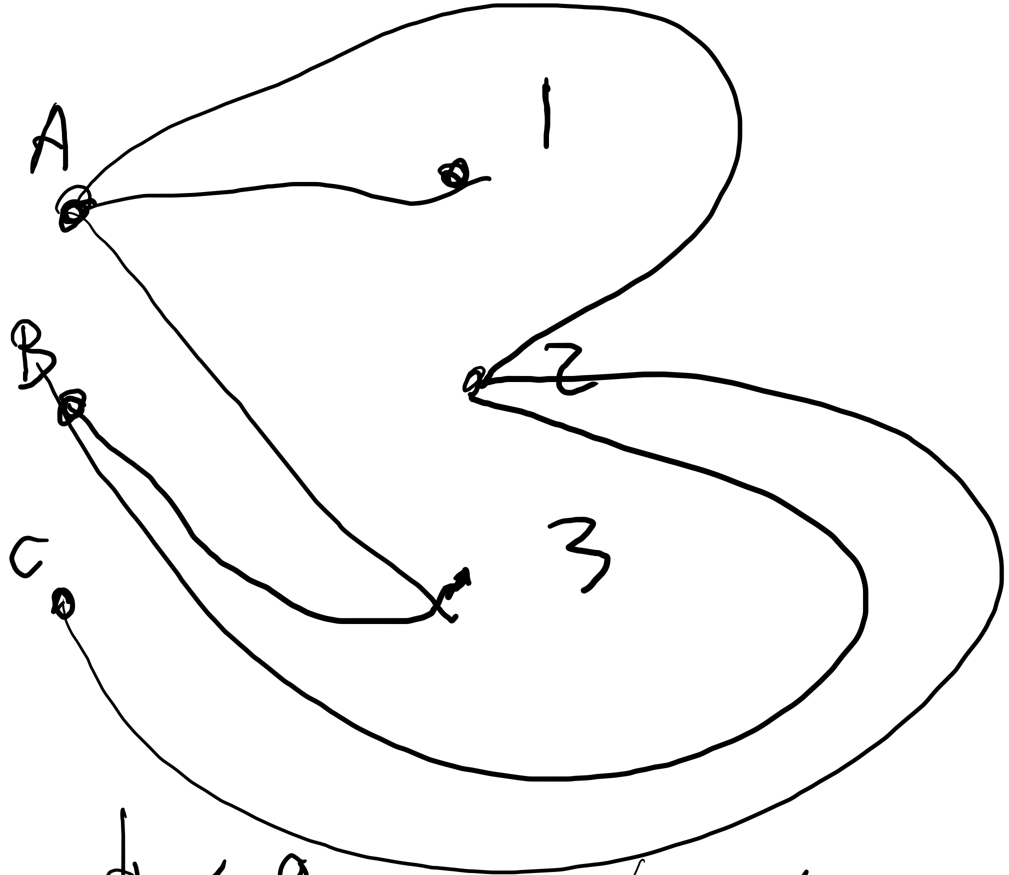
$$
\begin{aligned}
\delta &\le d(v_1) \\
+\,\delta &\le +\,d(v_2) \\
&\quad + \\
&\quad \vdots \\
+\,\delta &\le +\,d(v_\nu) \\
\hline
\delta\nu &\le \sum_{i=1}^{\nu} d(v_i) = 2\varepsilon
\end{aligned}
$$

$\delta\nu \le 6\nu - 12$

$\delta \le 6 - \frac{12}{\nu}$

$\delta < 6$

$\delta \le 5$



$4\phi \le 18 \qquad \phi \le \frac{9}{2} \qquad \phi \le 4$