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$$365 < 10.000$$

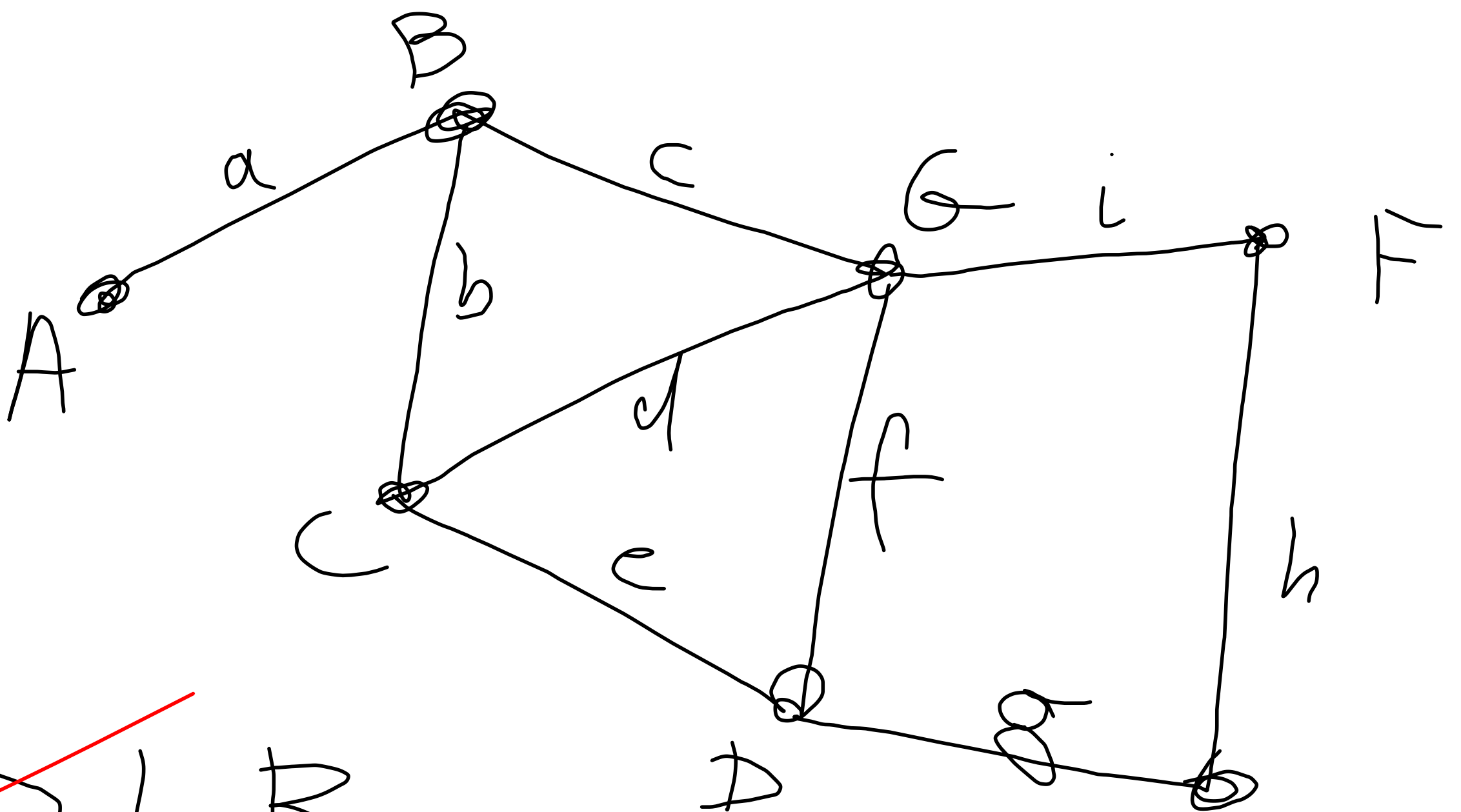
DEF - Two sets are said to have the same cardinality if there is at least one bijection between them.

DEF - Given sets X and Y , the cardinality of X is said to be less than or equal to the cardinality of Y

$|X| \leq |Y|$

if there is at least one injective map from X to Y .

$$\begin{aligned} \#X &< \#Y \\ \text{card } X &< \text{card } Y \end{aligned}$$



~~A e D h B~~

A a B c G c D b C

DEF - A (binary) relation R on a set X is called an **equivalence** relation if these three conditions hold:

1) R is **reflexive**: $\forall x \in X \quad x R x$ (read: x is in relation R with x)

2) R is **symmetric**: $\forall x, y \in X \quad x R y \Rightarrow y R x$

3) R is **transitive**: $\forall x, y, z \in X$
 $(x R y) \wedge (y R z) \Rightarrow x R z$

DEF - A **partition** of a set X is a set of sets $\mathcal{Q} = \{X_1, X_2, \dots, X_n\}$, where

1) X_1, X_2, \dots, X_n are subsets of X

2) if $X_i \neq X_j$, then $X_i \cap X_j = \emptyset$

3) $X_1 \cup X_2 \cup \dots \cup X_n = X$

DEF - Given an equivalence relation \mathcal{R} on X , the **equivalence class** of $x \in X$ is the set of all elements in relation \mathcal{R} with x

$$[x]_{\mathcal{R}} = \{y \in X \mid x \mathcal{R} y\}$$

THM - Given an equivalence relation \mathcal{R} on X ,
the set of its equivalence classes is a
partition of X .

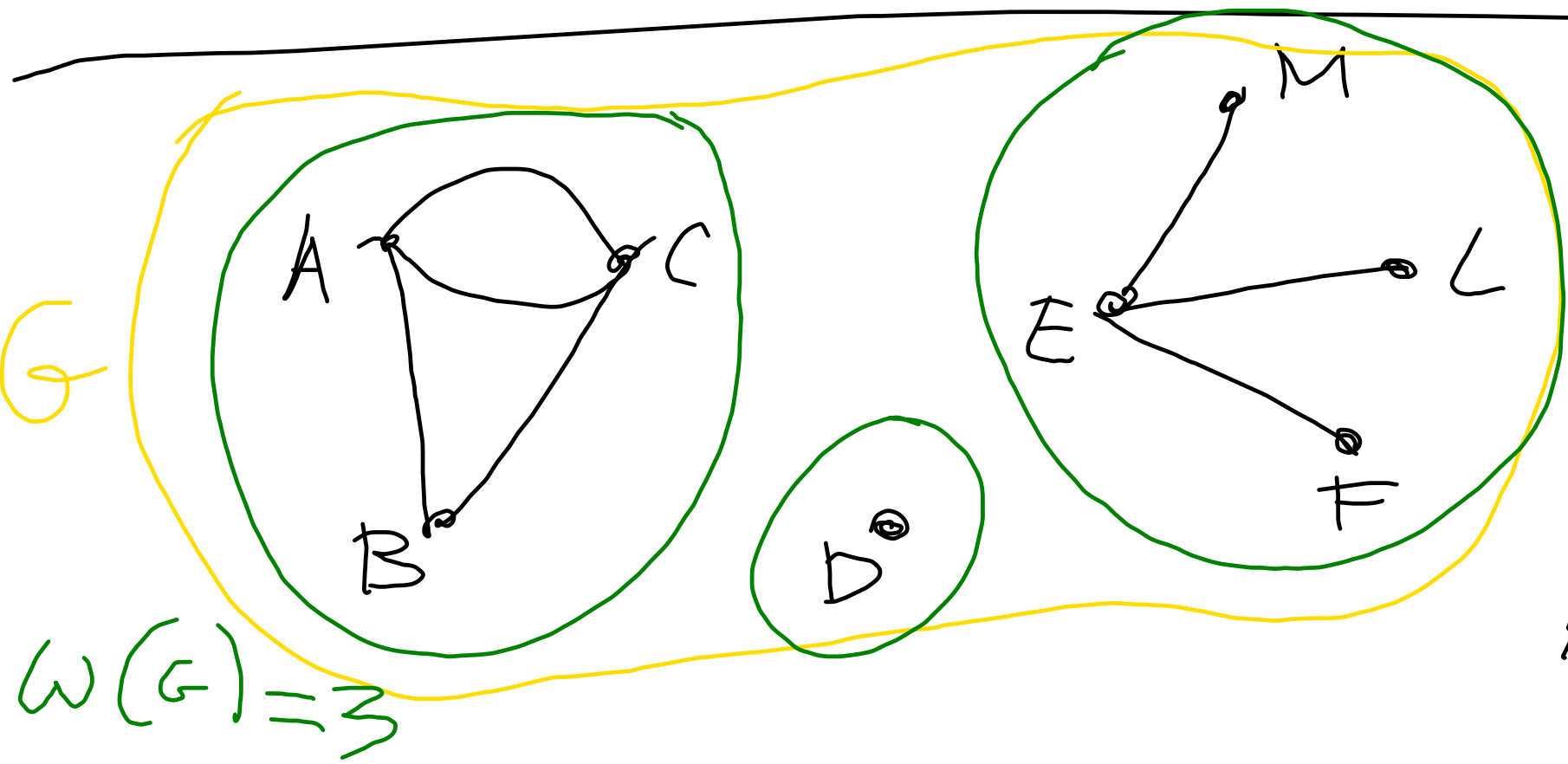
DEF - Given a graph G and a property \mathcal{P} which might hold for some of its subgraphs, we say that a subgraph H of G is **maximal with respect to** property \mathcal{P} if these two conditions hold:

- 1) H has property \mathcal{P}
- 2) no subgraph H' such that $H \subset H' \subseteq G$ has property \mathcal{P} .

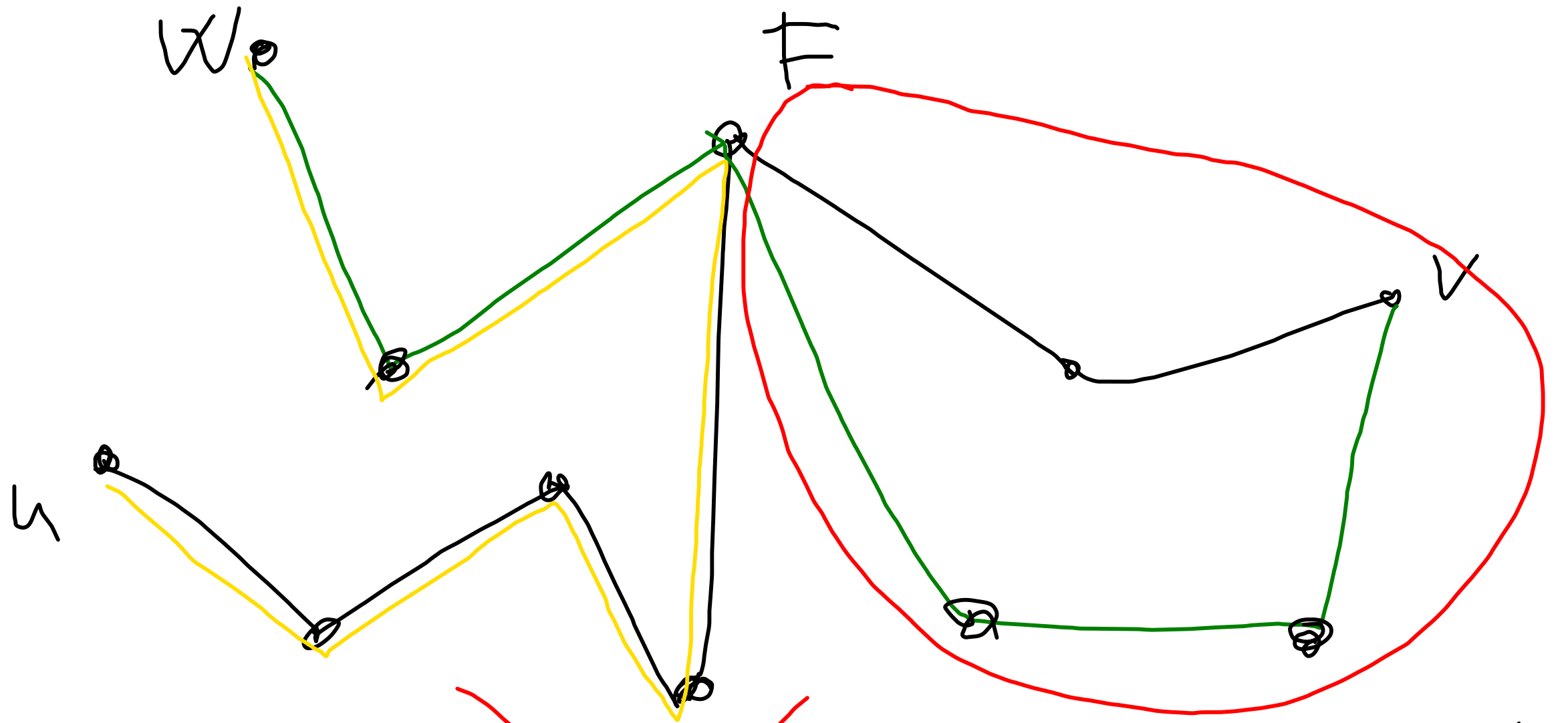
DEF - A graph G is said to be **connected** if $\forall u, v \in V(G)$ there is a (u, v) -path.

DEF - A **component** of a graph G is a maximal connected subgraph of G .

$G(\{C, E, F, L, M\})$



$G(\{E, L, M\})$ is connected but not a maximal connected graph, because $G(\{E, F, L, M\})$ is connected and contains it. $G(\{E, F, L, M\})$ is maximal connected.



$W_0 = v_0 v_1 \dots v_9 v_{10} \dots v_{26} v_{27} v_{28} \dots v_{100}$
 $W_1 = v_0 v_1 \dots v_9 \underbrace{\dots}_{F} v_{10} v_{28} \dots v_{100}$

DEF - Given a set X , a **distance** on X is a map $d: X \times X \rightarrow \mathbb{R}$ such that these three conditions hold:

$$1) \forall x, y \in X \quad d(x, y) \geq 0 \quad \text{and} \quad d = 0 \iff x = y$$

$$2) \forall x, y \in X \quad d(x, y) = d(y, x)$$

3) (triangle inequality)

$$\forall x, y, z \in X \quad d(x, z) \leq d(x, y) + d(y, z)$$