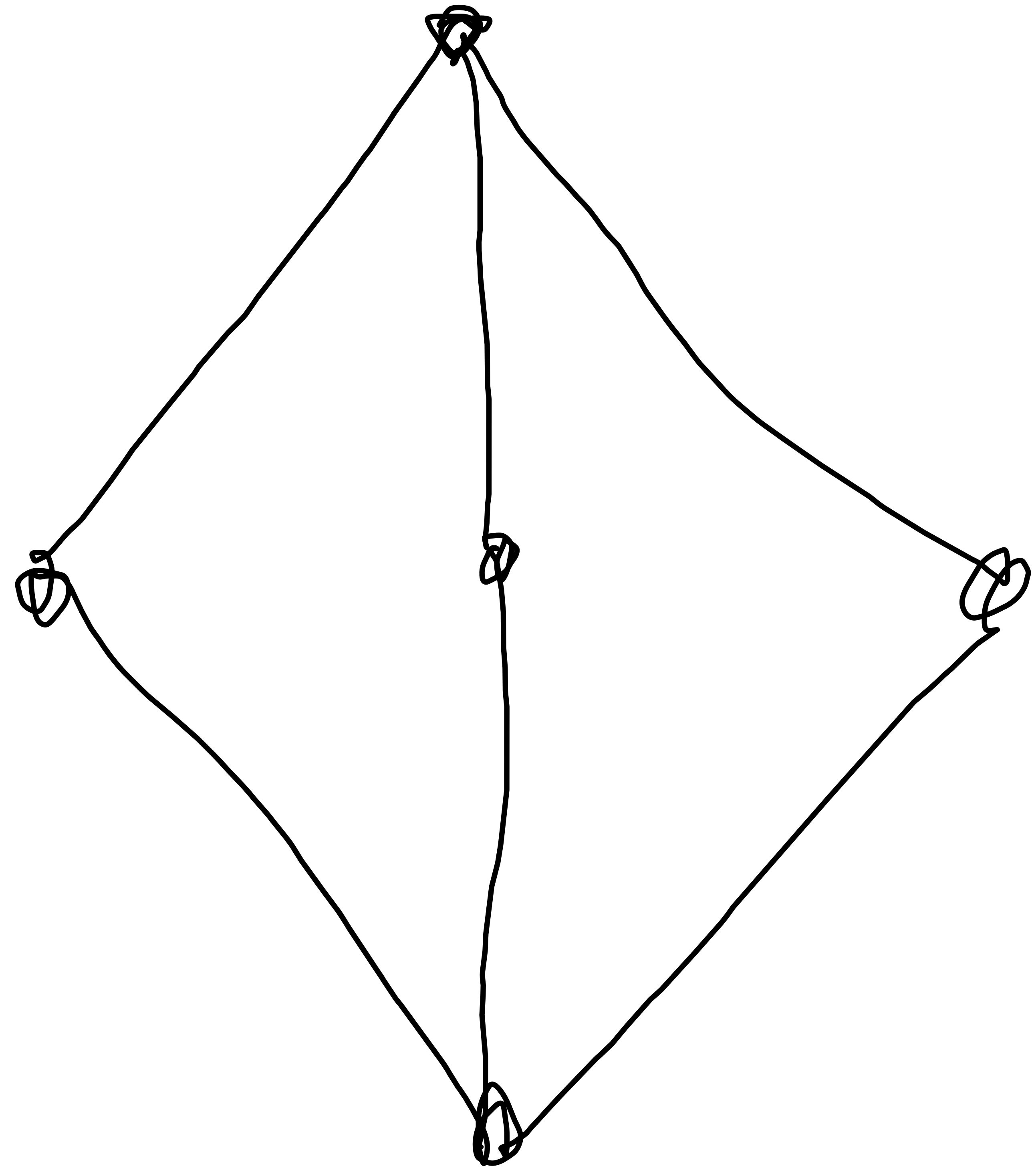
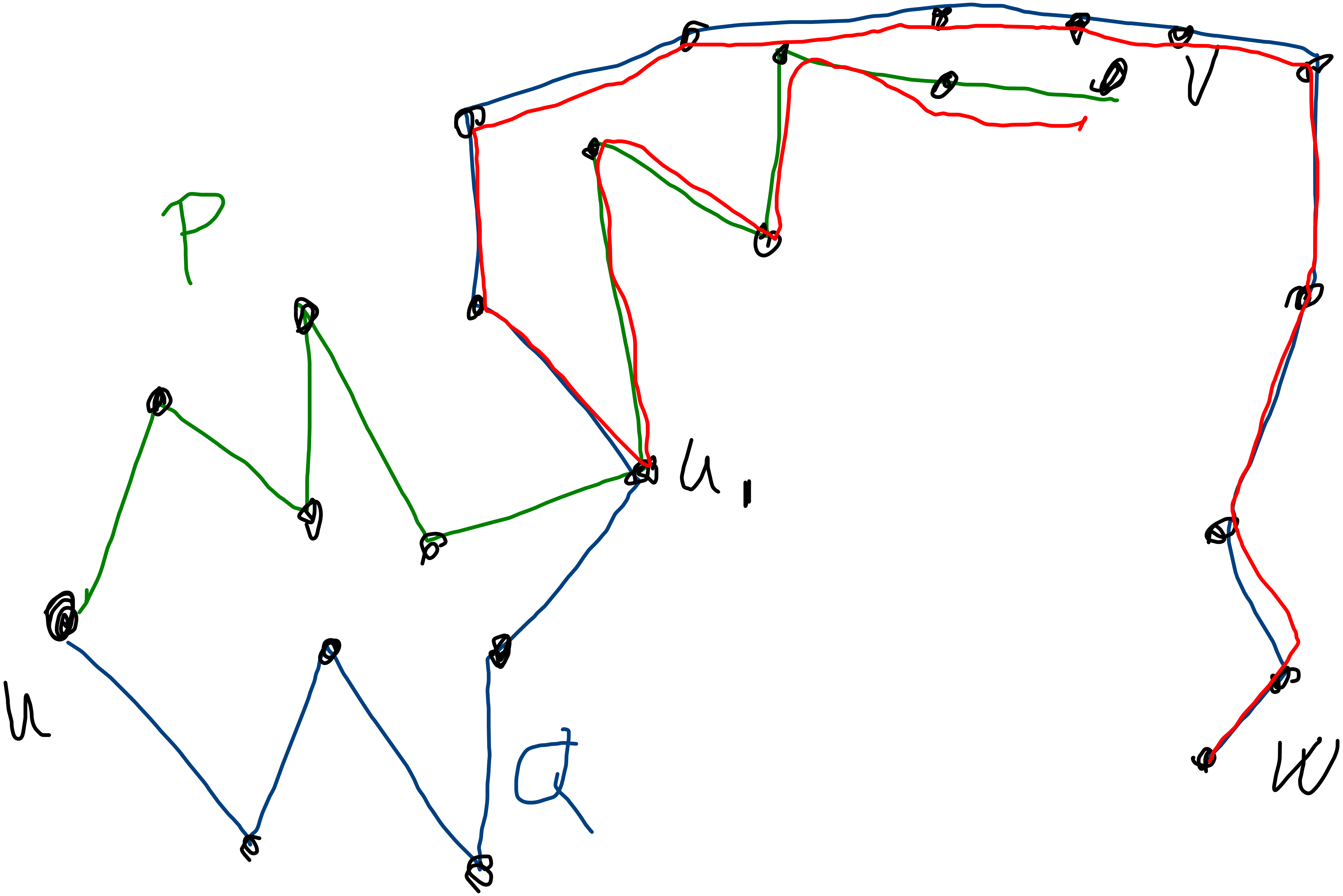



P
u
u_1
Q
v
w

$A \Rightarrow B$ is defined as equivalent to:

$(\neg A) \vee B$

A	B	$\neg A$	$\overset{A \Rightarrow B}{(\neg A) \vee B}$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

DEF **Counterpositive** of the implication $A \Rightarrow B$:

$C \Longrightarrow D$ $(\neg C) \vee D$

it's the implication $(\neg B) \Rightarrow (\neg A)$.

THM Every implication is equivalent to its counterpositive.

PROOF $(\neg B) \Rightarrow (\neg A)$

is equiv. to $(\neg(\neg B)) \vee (\neg A)$

is equiv. to $B \vee (\neg A)$

is equiv. to $(\neg A) \vee B$

is equiv. to $A \Rightarrow B$

"Proof by induction"

It is a method resembling the recursion procedures, but in proving a statement.

You have to prove a property P depending on a natural number n. This is the induction scheme:

1) Inductive premise: you prove P for a low n (generally 0, or 1, or 2...)

2) Inductive step: out of the inductive hypothesis (P is true for k-1) you prove the inductive thesis (P is true for k).

PROP – The number $S(n)$ of edges in K_n is $S(n) = n(n-1)/2$.

PROOF –

Ind. premise: K_2

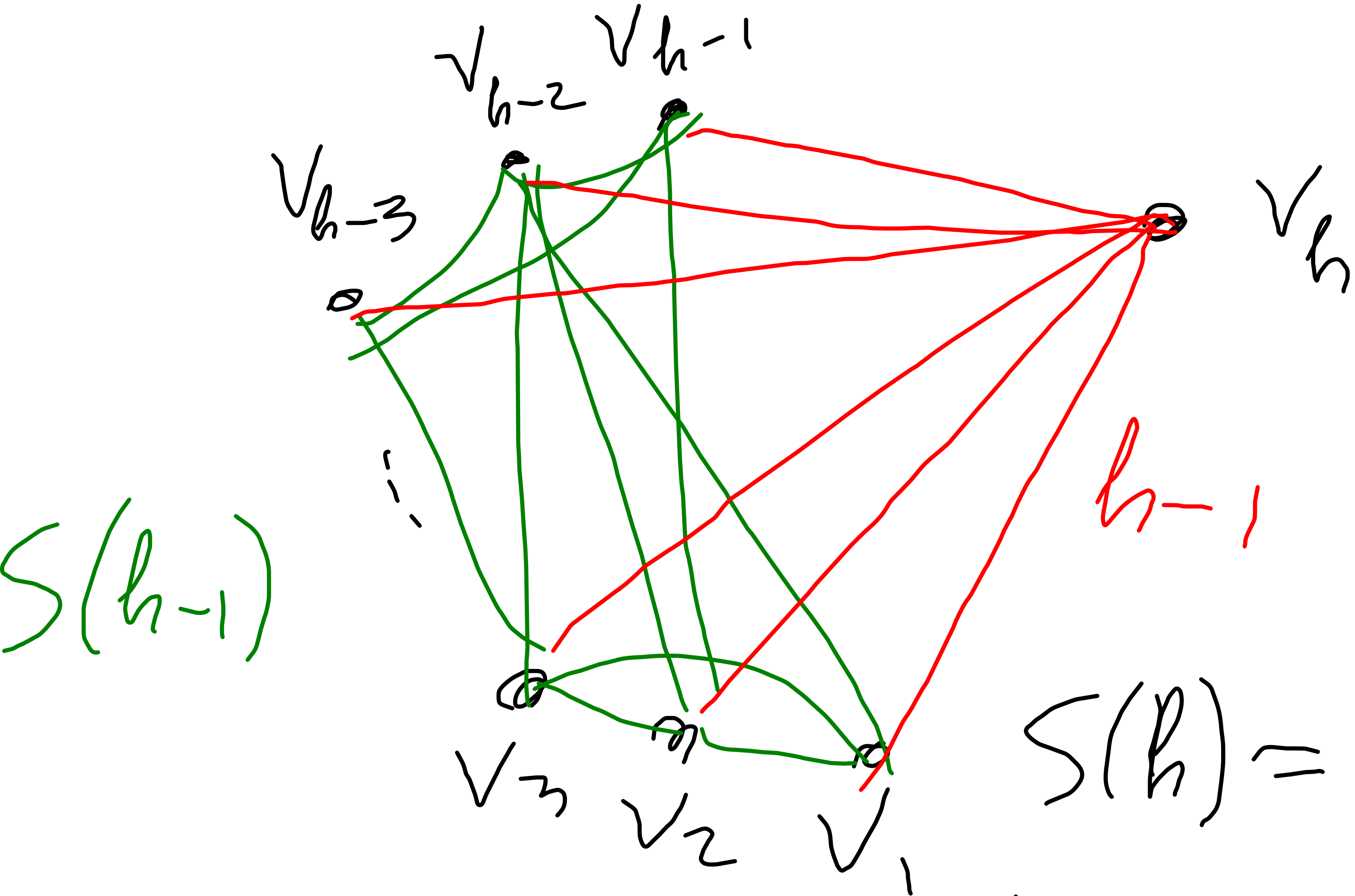
$S(2) = 1$ ✓

$n(n-1)/2 = 2 \cdot 1/2 = 1$

Ind. step:

Ind. hypothesis: $S(n-1) = (n-1)(n-1-1)/2$

Ind. Thesis: $S(n) = n(n-1)/2$



$S(n) = S(n-1) + n-1$

now apply the Ind. Hyp. : $S(n-1) = \frac{(n-1)(n-2)}{2}$

$$S(n) = \frac{(n-1)(n-2)}{2} + n-1 =$$

$$= \frac{(n-1)(n-2)+2(n-1)}{2} = \frac{(n-1)(n-2+2)}{2} = \frac{n(n-1)}{2}$$

✓

$\varepsilon(G_1) = \nu(G_1) - 1 \quad \varepsilon(G_2) = \nu(G_2) - 1$

$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + 1 =$

$= \nu(G_1) \cancel{-1} + \nu(G_2) - 1 \cancel{+1} =$

$= \nu(G_1) + \nu(G_2) - 1 =$

$= \nu(G) - 1$

✓