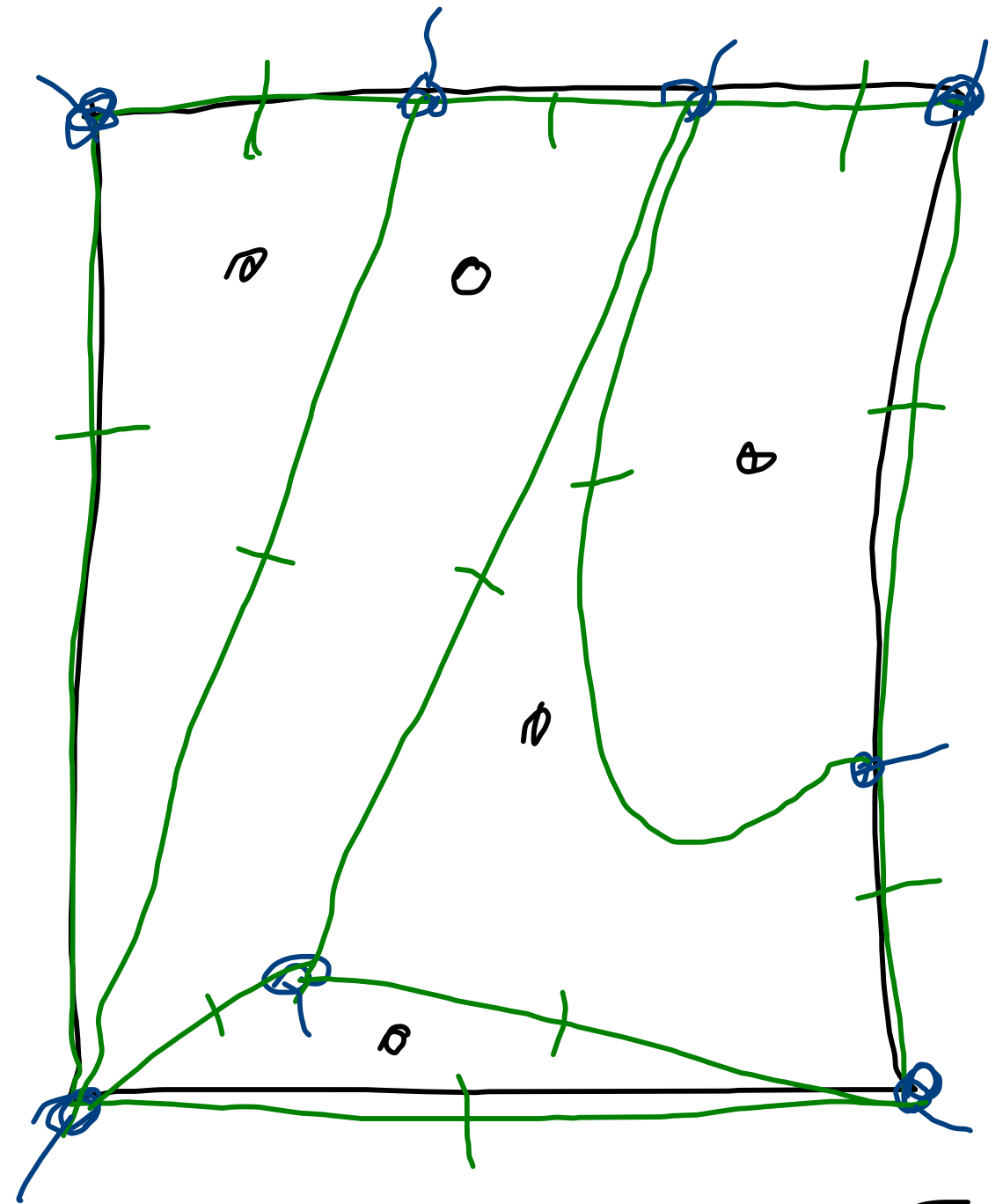


$$7 \quad 10 \quad 4$$

$$7 + 4 - 10 = 1$$



$$8 \quad 12 \quad 5$$

$$8 + 5 - 12 = 1$$

G tree v # vertices
 $v-1$ # edges
 1 # faces

$$v - (v-1) + 1 = \cancel{v} - \cancel{v} + 1 + 1 = 2$$

$$2 = v(G) - (\varepsilon(G) - 1) + (\phi(G) - 1) =$$

$$v(G) - \varepsilon(G) + \cancel{1} + \phi(G) - \cancel{1}$$

$$\varphi \leq \frac{2\varepsilon}{3}$$

$$2 = \nu - \varepsilon + \varphi \leq \nu - \varepsilon + \frac{2\varepsilon}{3} = \frac{3\nu - 3\varepsilon + 2\varepsilon}{3}$$

$$3\nu - \varepsilon \geq 6$$

$$\varepsilon \leq 3\nu - 6$$

$$\frac{\delta \nu}{\nu} \leq 6\nu - 12$$

$$\delta \leq 6 - \frac{12}{\nu} \neq 0$$
$$\delta \leq 5$$