No 111

No 011

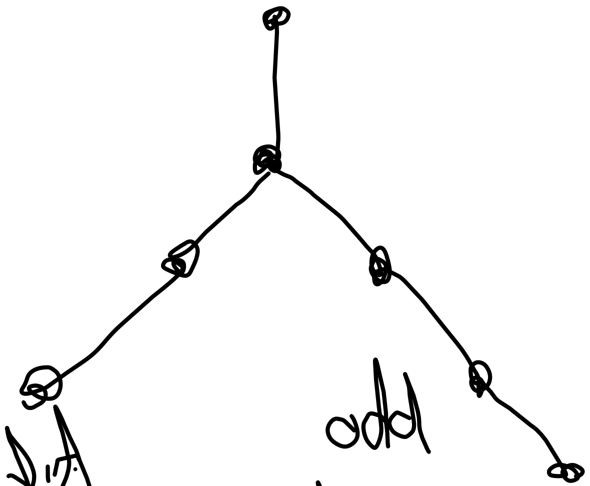
No 101

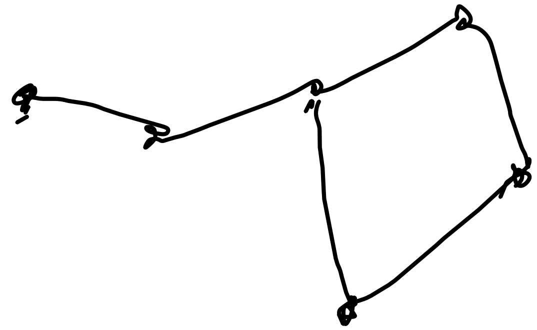
No 110

No 001

No 010

No 100

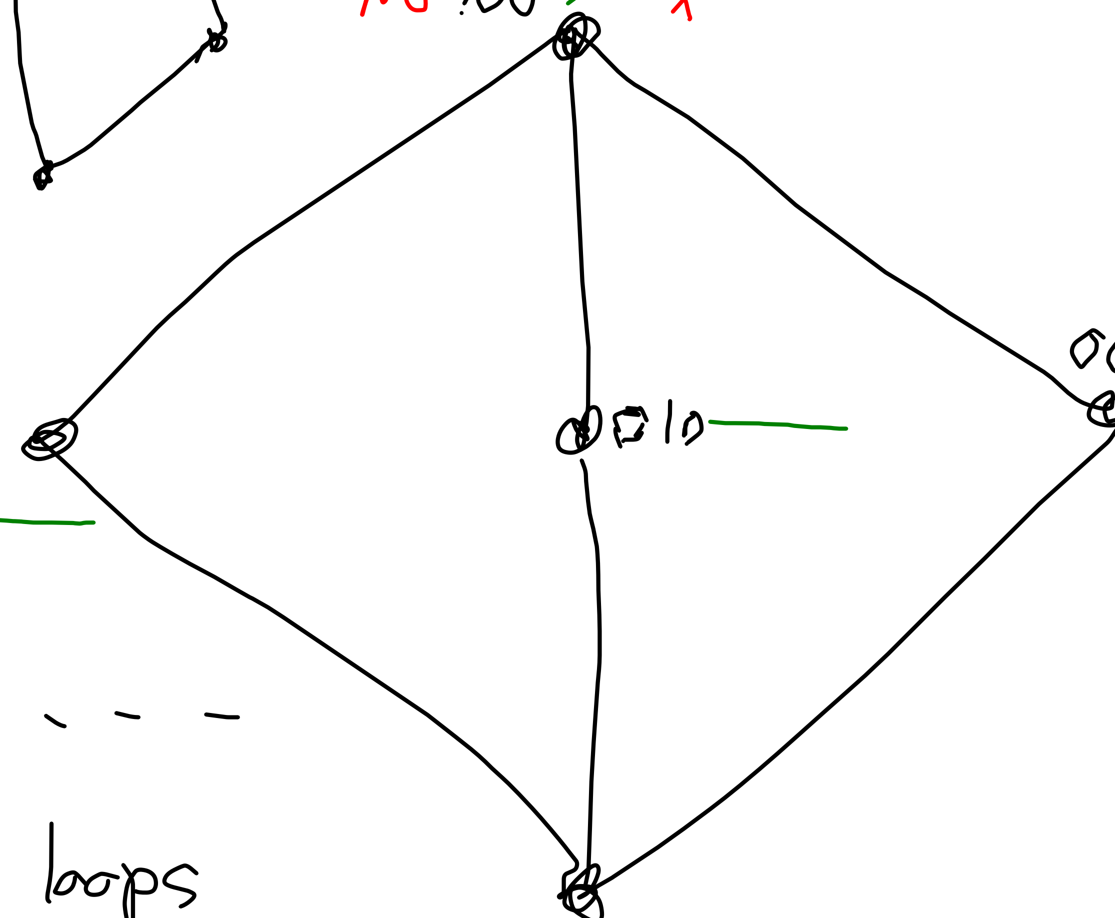




odd

even odd

$(d_1 \quad d_2 \; \dots \; d_k$



100

010

001

000

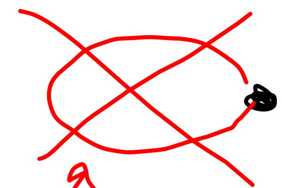
v_1 $\quad$ v_2 $\quad$ v_k

$d_1/2$ loops $\quad (d_2-1)/2$ loops $\quad (d_k-1)/2$ loops

Thm - If G is a tree, then $\varepsilon(G) = \nu(G) - 1$

PROOF - By induction on the number of vertices

Inductive premise: $\nu = 1$



cycle

Trivial

$\Rightarrow \varepsilon = 0 = \nu - 1$

Inductive hypothesis: statement is true for all trees with less then n vertices

Inductive thesis: statement is true for all trees with n vertices

Proof of the inductive thesis:

G a tree with $v(G) = n \geq 2$. It must contain at least one edge $e = uv$. The edge e is itself the only path from u to v. Therefore $G - e$ must be disconnected. Actually, $G - e$ is made of 2 components, one containing u, call it G_1, one containing v, call it G_2. G_1 and G_2 are connected and acyclic, so they are trees. Each of G_1, G_2 has less then n vertices, so the statement applies to them:

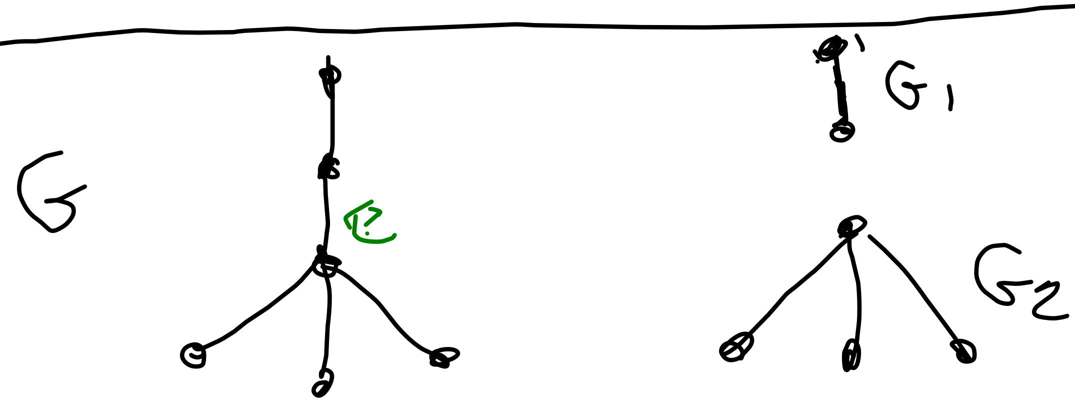
$$\varepsilon(G_1) = \nu(G_1) - 1 \qquad \varepsilon(G_2) = \nu(G_2) - 1$$

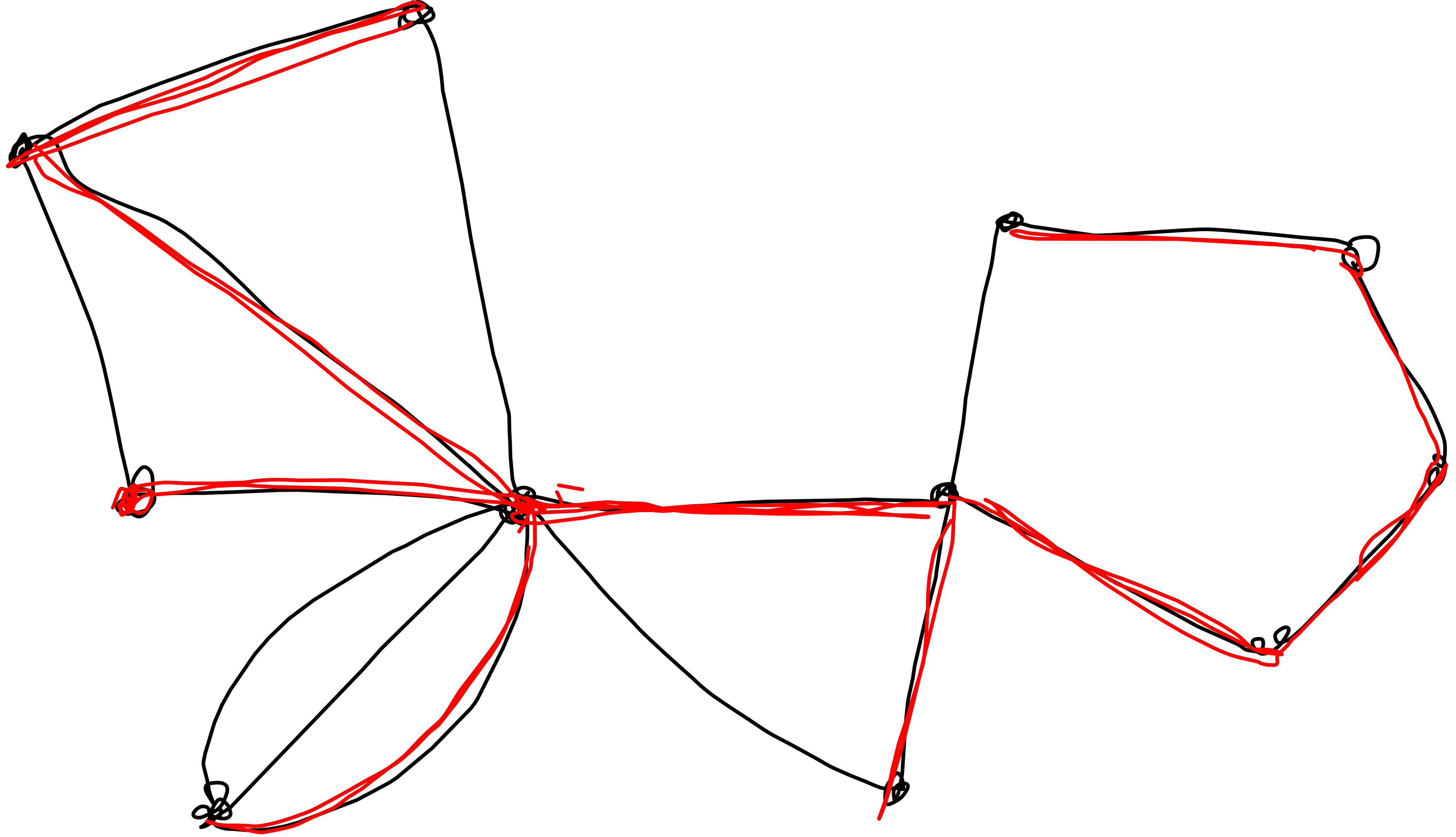
$$\nu(G) = \nu(G_1) + \nu(G_2)$$

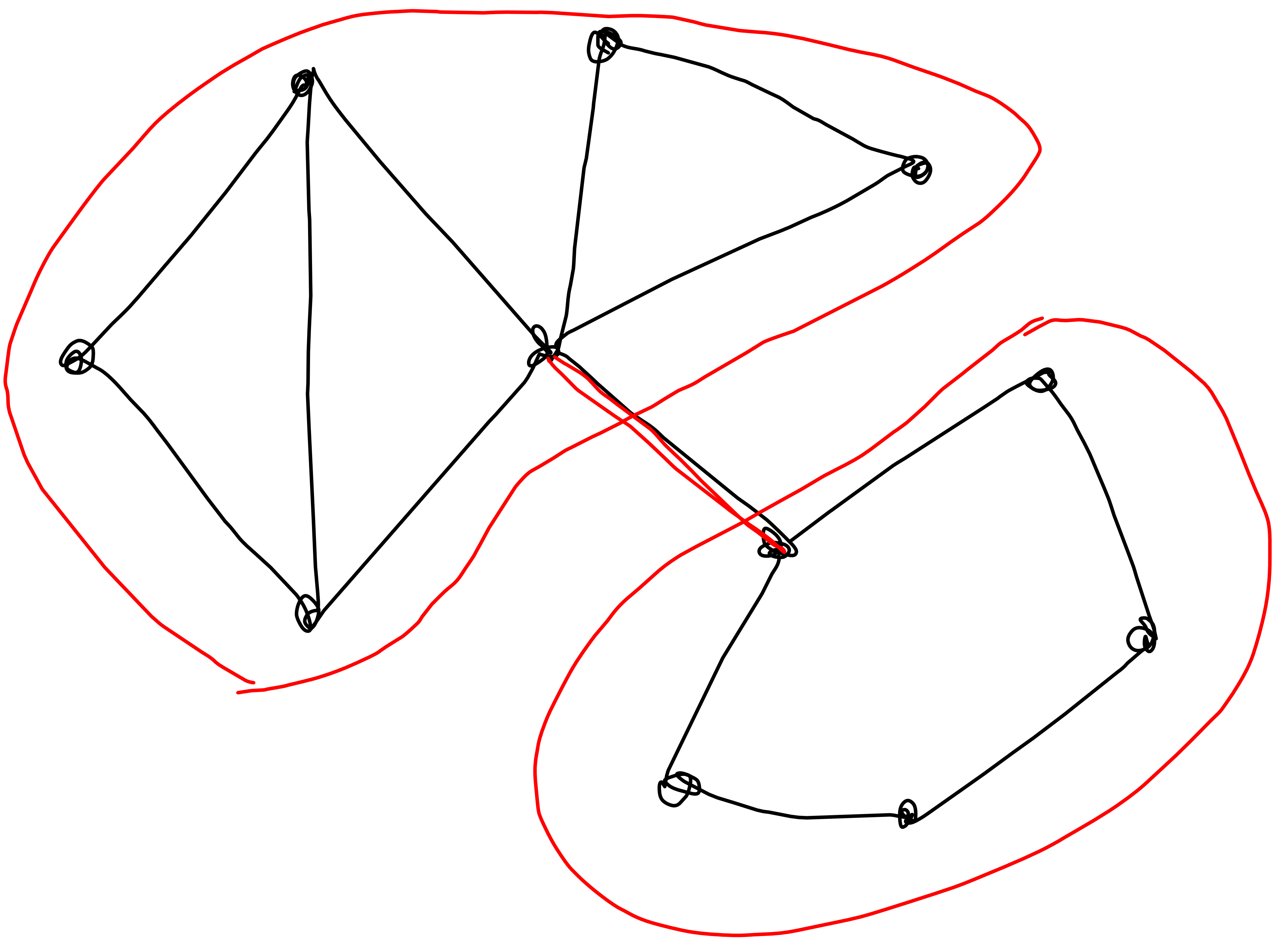
$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + 1$$

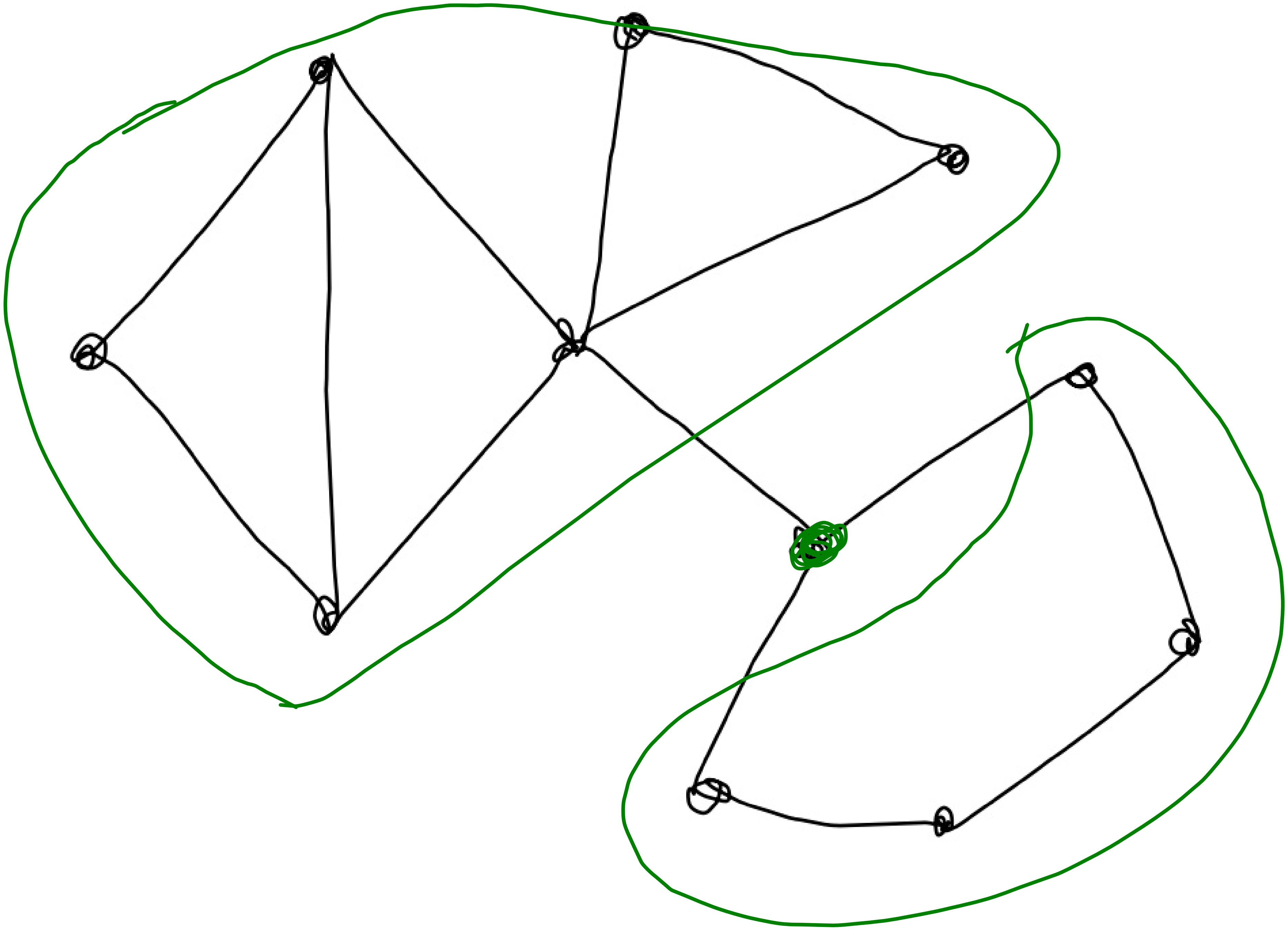
$$\varepsilon(G) = \varepsilon(G_1) + \varepsilon(G_2) + 1 =$$

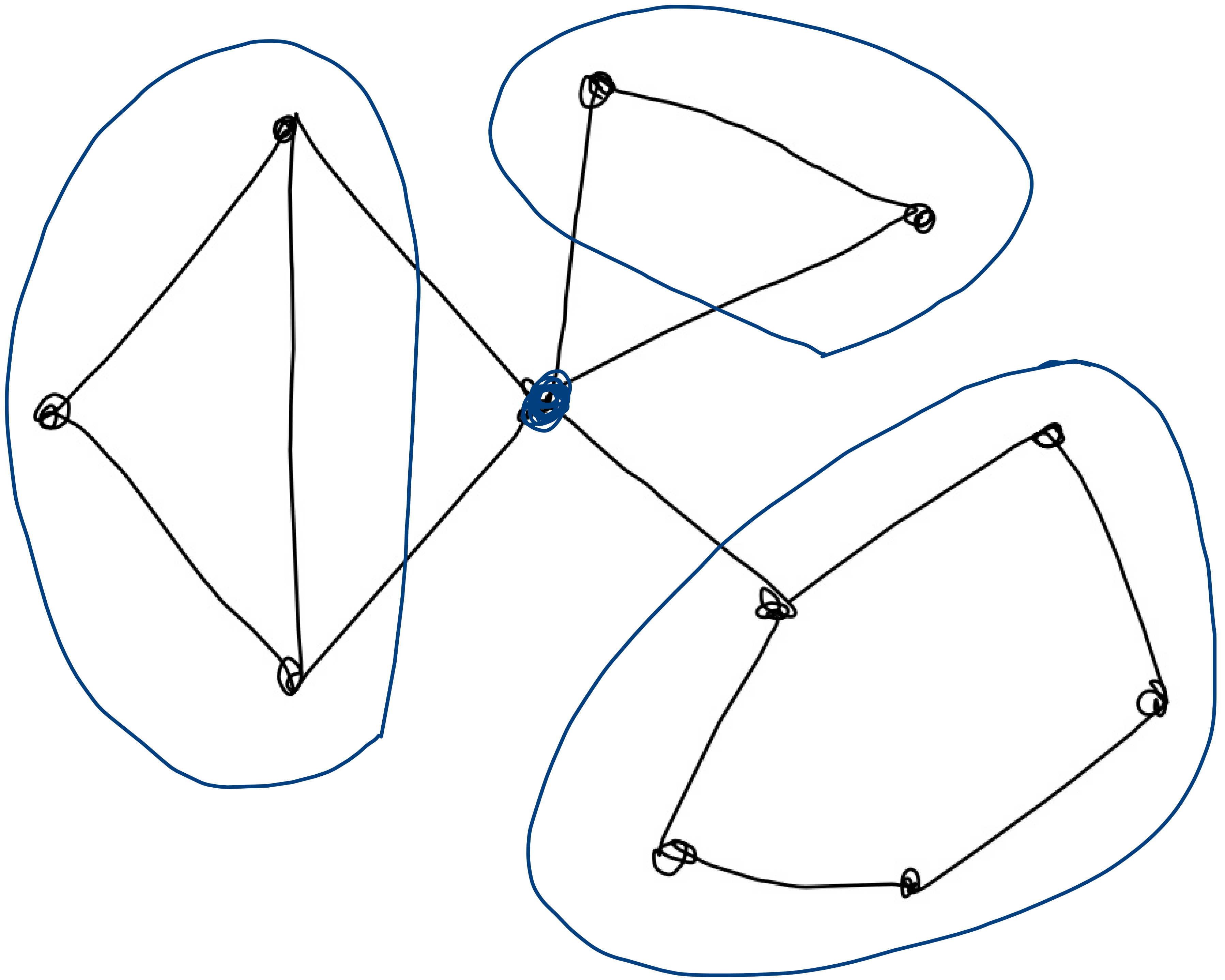
$$= \nu(G_1) \cancel{-1} + \nu(G_2) - 1 \cancel{+1} = \nu(G) - 1$$

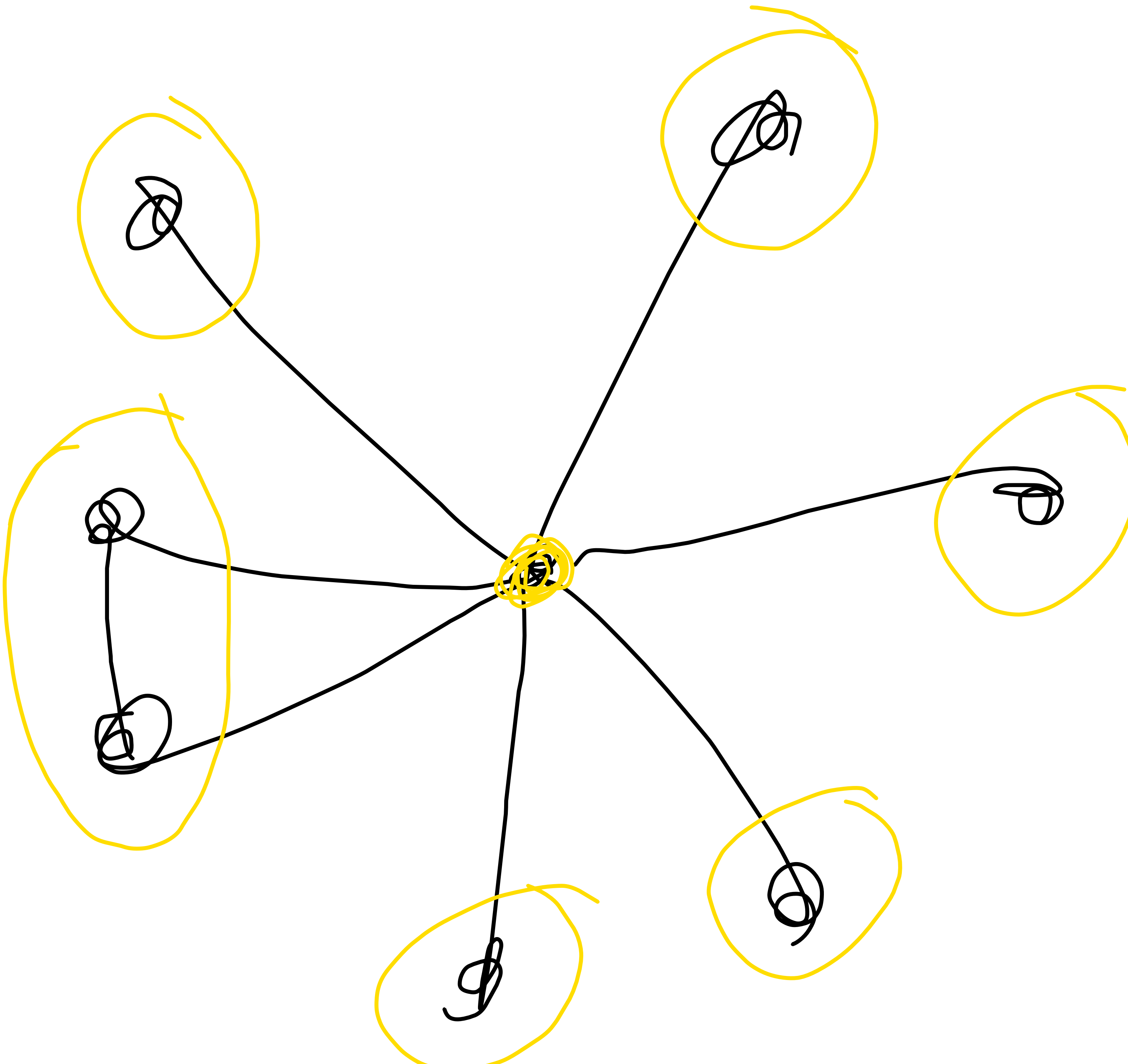


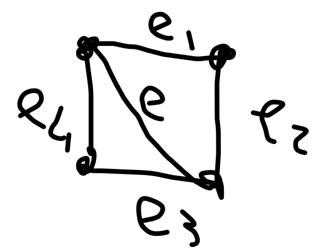




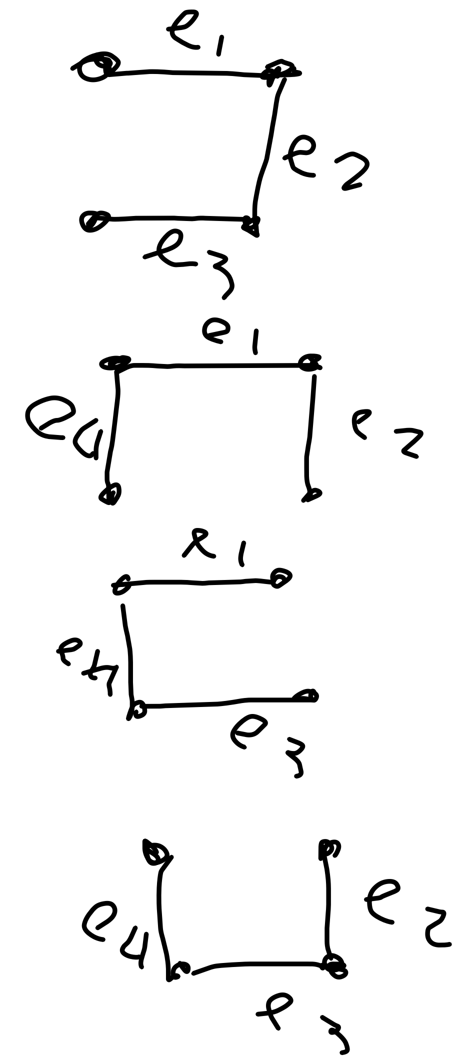




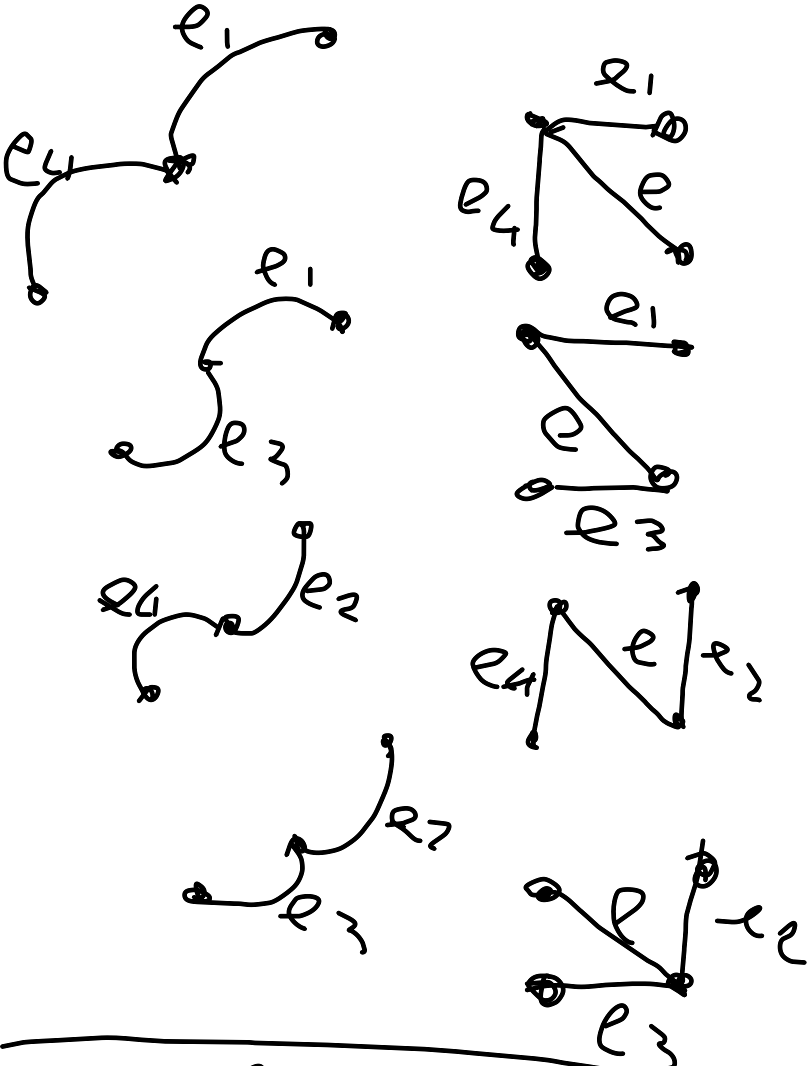


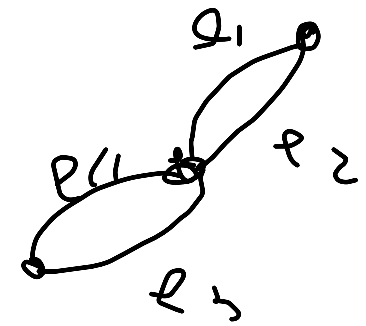


G

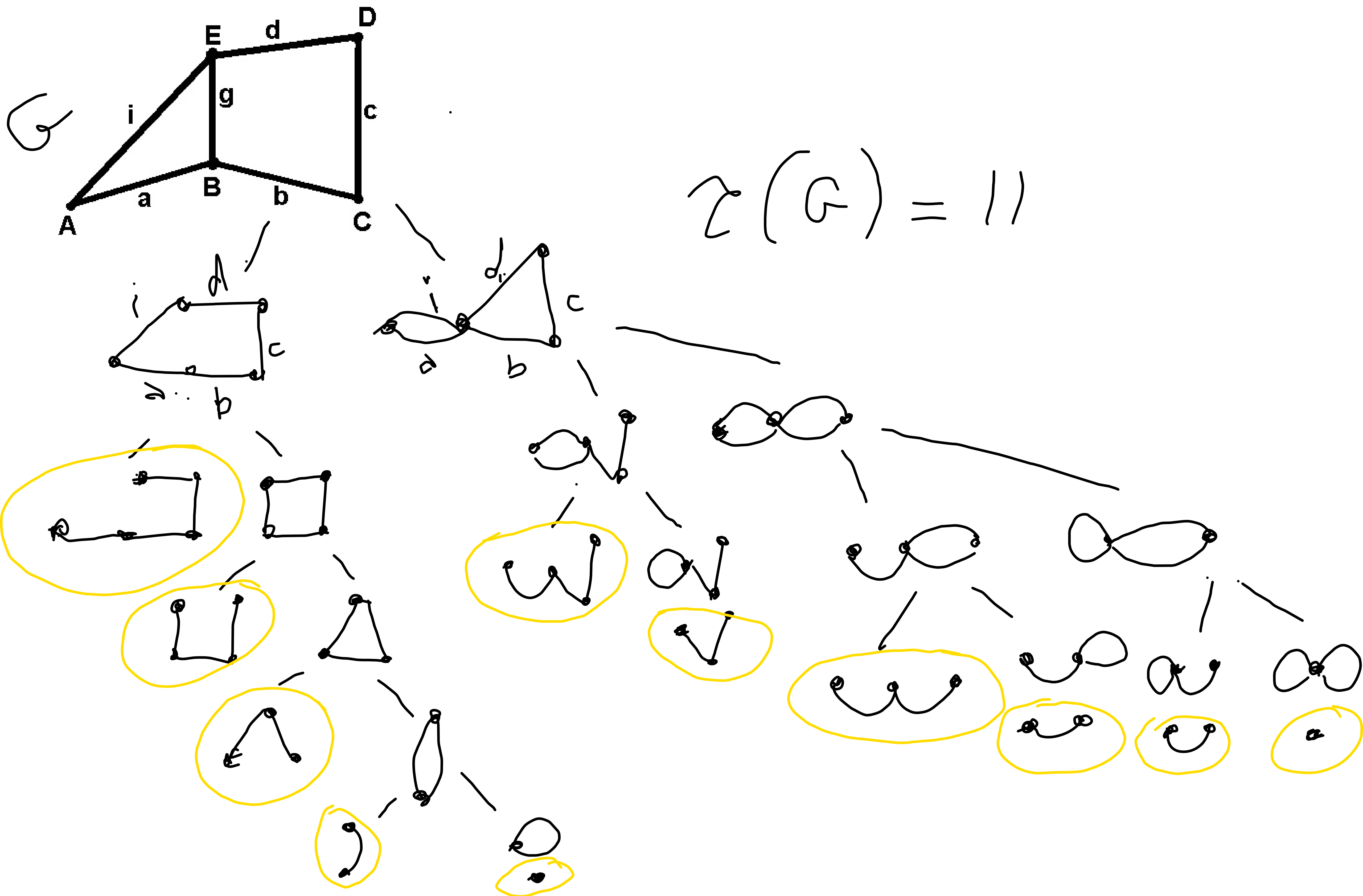


G − e



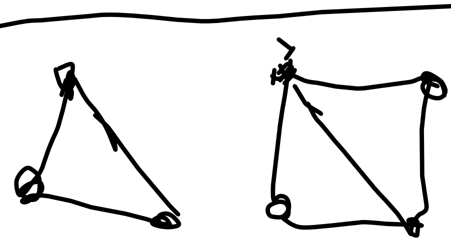


G · e

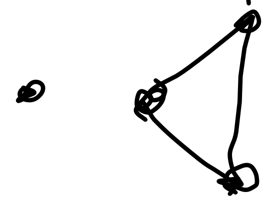


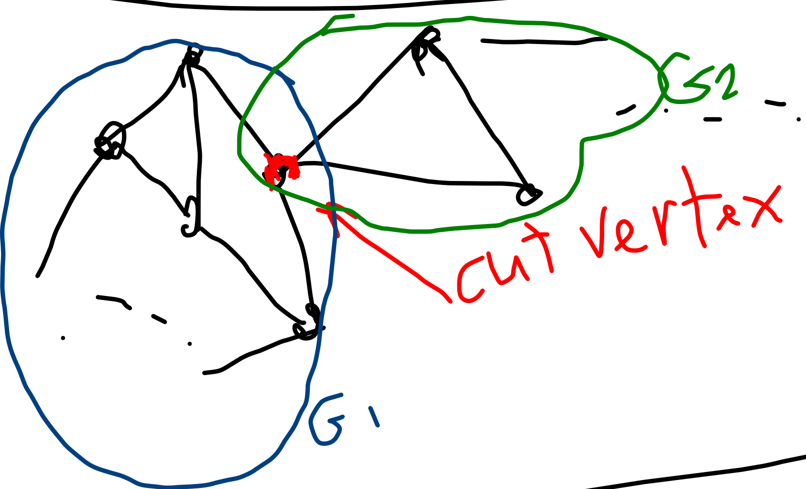
$\tau(G) = 11$

Shortcuts in the computation of τ:

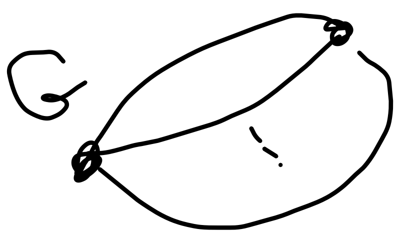


$\tau = 0$



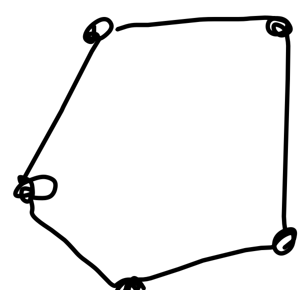


$\tau(G) = \tau(G_1) \cdot \tau(G_2)$



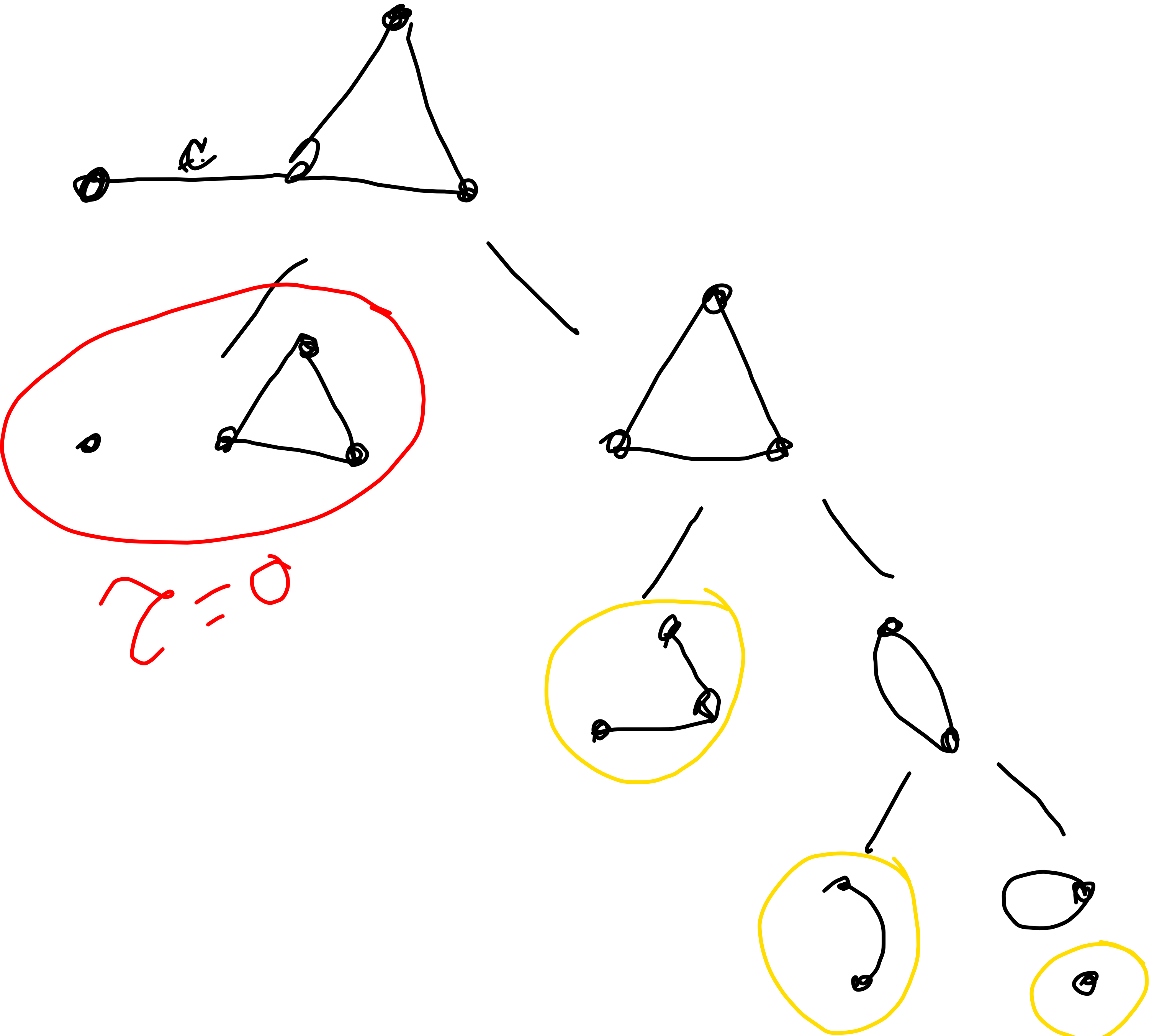
k multiple edges

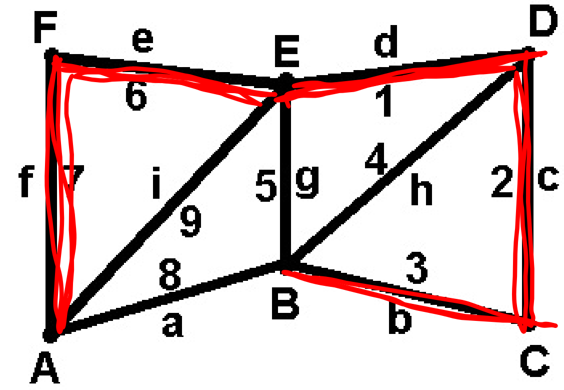
$\tau(G) = k$

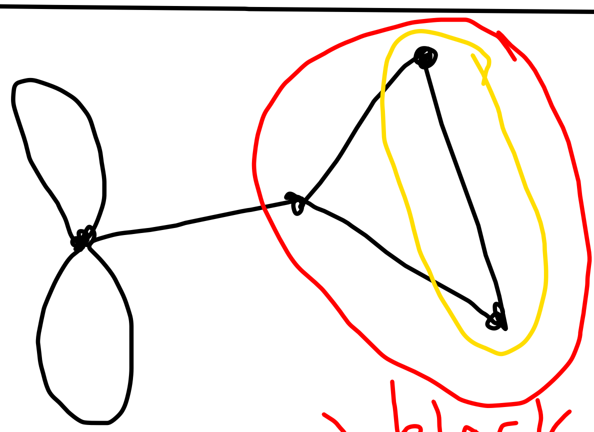


cycle of k edges

$\tau(G) = k$

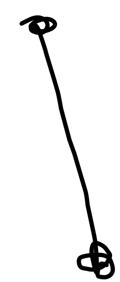

e
τ = 0

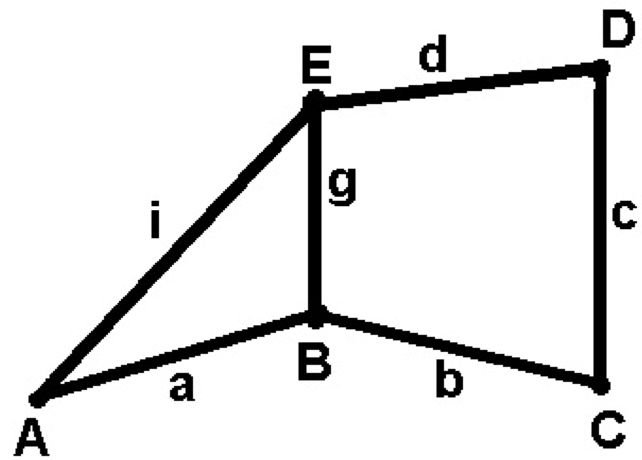


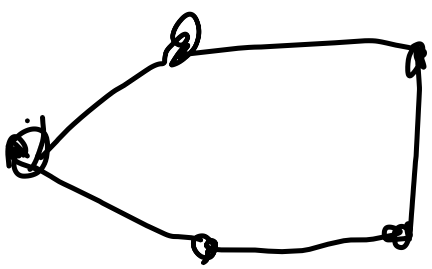


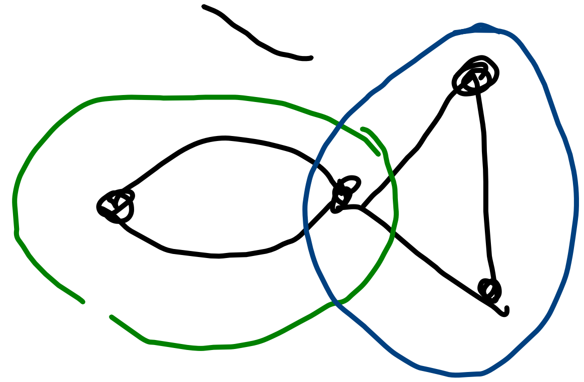
why NOT a block of G

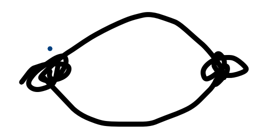
a block of G



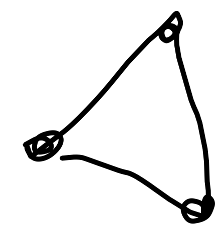








$\tau = 2$



$\tau = 3$

$\tau = 5$

$\tau = 2 \cdot 3 = 6$

$\tau(G) = 5 + 6 = 11$