

$$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \gamma^0 = 2$$

$$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

$$\gamma^1 = 1 \quad H_0 \cong \mathbb{Z} \quad \beta_0 = \alpha^0 - \gamma^0 - \gamma^{-1} = 3 - 2 - 0 = 1$$

$$H_1 \cong H_2 \cong 0 \quad \beta_1 = \alpha^1 - \gamma^1 - \gamma^0 = 3 - 1 - 2 = 0$$

$$\beta_2 = \alpha^2 - \gamma^2 - \gamma^1 = 1 - 0 - 1 = 0$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \gamma^0 = 1$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 \\ 0 & -2 \\ 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix} \quad \gamma^1 = 2$$

$$\beta_0 = \alpha^0 - \gamma^0 - \gamma^1 = 2 - 1 - 0 = 1$$

$$\beta_1 = \alpha^1 - \gamma^1 - \gamma^0 = 3 - 2 - 1 = 0$$

$$\beta_2 = \alpha^2 - \gamma^2 - \gamma^1 = 2 - 0 - 2 = 0$$

$$H_0 \cong \mathbb{Z}$$

$$H_1 \cong \mathbb{Z}_2$$

$$H_2 \cong 0$$

tors.

in par.

$$\beta_n = \alpha^n - \cancel{\gamma^n} - \cancel{\gamma^{n-1}}$$

$$\beta_{n-1} = \alpha^{n-1} - \cancel{\gamma^{n-1}} - \cancel{\gamma^{n-2}}$$

$$\beta_2 = \alpha^2 - \cancel{\gamma^2} - \cancel{\gamma^1}$$

$$\beta_1 = \alpha^1 - \cancel{\gamma^1} - \cancel{\gamma^0}$$

$$\beta_0 = \alpha^0 - \cancel{\gamma^0} - \cancel{\gamma^{-1}}$$