



$$M = M_1 \cup D^n$$

$$M_1 \cap D^n = S^{n-1}$$

$$k \leq n-2$$

$$\begin{array}{ccccccc} \tilde{H}_k(S^{n-1}) & \longrightarrow & \tilde{H}_k(M_1) \oplus \tilde{H}_k(D^n) & \longrightarrow & \tilde{H}_k(M) & \longrightarrow & \tilde{H}_{k-1}(S^{n-1}) \\ \circlearrowleft & & \circlearrowleft & & \circlearrowleft & & \circlearrowleft \end{array}$$

$$\tilde{H}_k(M_1) \cong \tilde{H}_k(M)$$

$$\begin{array}{ccccccc} \tilde{H}_k(S^{n-1}) & \longrightarrow & \tilde{H}_k(M_1) \oplus \tilde{H}_k(N_1) & \longrightarrow & \tilde{H}_k(M \# N) & \longrightarrow & \tilde{H}_{k-1}(S^{n-1}) \\ \circlearrowleft & & \circlearrowleft & & \circlearrowleft & & \circlearrowleft \end{array}$$

$$\tilde{H}_k(M \# N) \cong \tilde{H}_k(M_1) \oplus \tilde{H}_k(N_1) \cong \tilde{H}_k(M) \oplus \tilde{H}_k(N)$$

