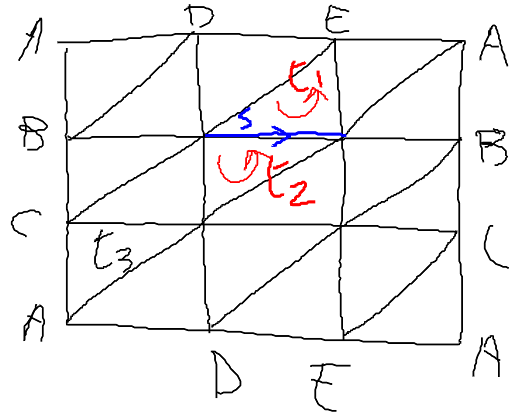
$t_1^*, t_2^* \in \mathcal{G}^2$



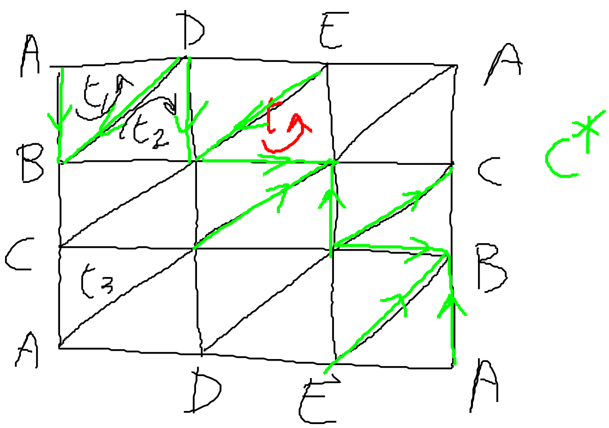
$\delta s^* \in \mathcal{G}^2$

$\langle t_3, \delta s^* \rangle = \langle \partial t_3, s^* \rangle = 0$

$\langle t_1, \delta s^* \rangle = \langle \partial t_1, s^* \rangle = 1$

$\langle t_2, \delta s^* \rangle = \langle \partial t_2, s^* \rangle = -1$

$\Rightarrow \delta s^* = t_1^* - t_2^*$



$\langle t_1, \delta c^* \rangle = \langle \partial t_1, c^* \rangle =$
$= 0 + 1 - 1 = 0$

$\langle t_2, \delta c^* \rangle = 0$

$\langle t_3, \delta c^* \rangle = 0 + 0 + 0$

$\vdots$

$\langle t, \delta c^* \rangle = 0 + 1 + 1 = 2$

$\langle t, t^* \rangle = 1$

$\Downarrow$

$\delta c^* = 2 t^*$

$$(p,q) \longmapsto (p,\bar{q})$$

$$0 \to \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z}\oplus\mathbb{Z} \xrightarrow{\beta} \mathbb{Z}\oplus\mathbb{Z}_k \to 0$$

$$q \longmapsto (0,kq)$$

$\mathrm{Hom}(-,\mathbb{Z})$

$$\mathrm{Hom}(\mathbb{Z},\mathbb{Z}) \cong \mathbb{Z}$$

$$f'': q \mapsto hq \longmapsto h$$

$$\mathrm{Hom}(\mathbb{Z}\oplus\mathbb{Z},\mathbb{Z}) \cong \mathbb{Z}\oplus\mathbb{Z}$$

$$f': (p,q) \mapsto hp + h'q \longmapsto (h,h')$$

$$\mathrm{Hom}(\mathbb{Z}\oplus\mathbb{Z}_k,\mathbb{Z}) \cong \mathbb{Z}$$

$$f: (p,\bar{q}) \mapsto hp \longmapsto h$$

$$(p,q) \longmapsto (p,\bar{q})$$

$$0 \to \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z}\oplus\mathbb{Z} \xrightarrow{\beta} \mathbb{Z}\oplus\mathbb{Z}_k \to 0$$

$$q \longmapsto (0,kq)$$

$$\mathrm{Hom}(\mathbb{Z},\mathbb{Z}) \hookleftarrow \mathrm{Hom}(\mathbb{Z}\oplus\mathbb{Z},\mathbb{Z}) \hookleftarrow \mathrm{Hom}(\mathbb{Z}\oplus\mathbb{Z}_k,\mathbb{Z}) \leftarrow 0$$

$$(f\beta)(p,q) = hp \longleftarrow f(p,\bar{q}) = hp$$

$$(f'\alpha)(q) = h'kq \longleftarrow f'(p,q) = hp + h'q$$

$$\begin{tikzcd}
0 & \mathbb{Z}_k \arrow[l] \arrow[d, equal] & \mathbb{Z} \arrow[l] & \mathbb{Z}\oplus\mathbb{Z} \arrow[l] & \mathbb{Z} \arrow[l] & 0 \arrow[l] \\
 & \operatorname{Coker}(\alpha, 1_{\mathbb{Z}}) & & (h,0) & h \arrow[l, mapsto] & \\
 & & h'k & (h,h') \arrow[l, mapsto] & &
\end{tikzcd}$$

$$\operatorname{Ext}(\mathbb{Z}\oplus\mathbb{Z}_k, \mathbb{Z}) \cong \mathbb{Z}_k$$