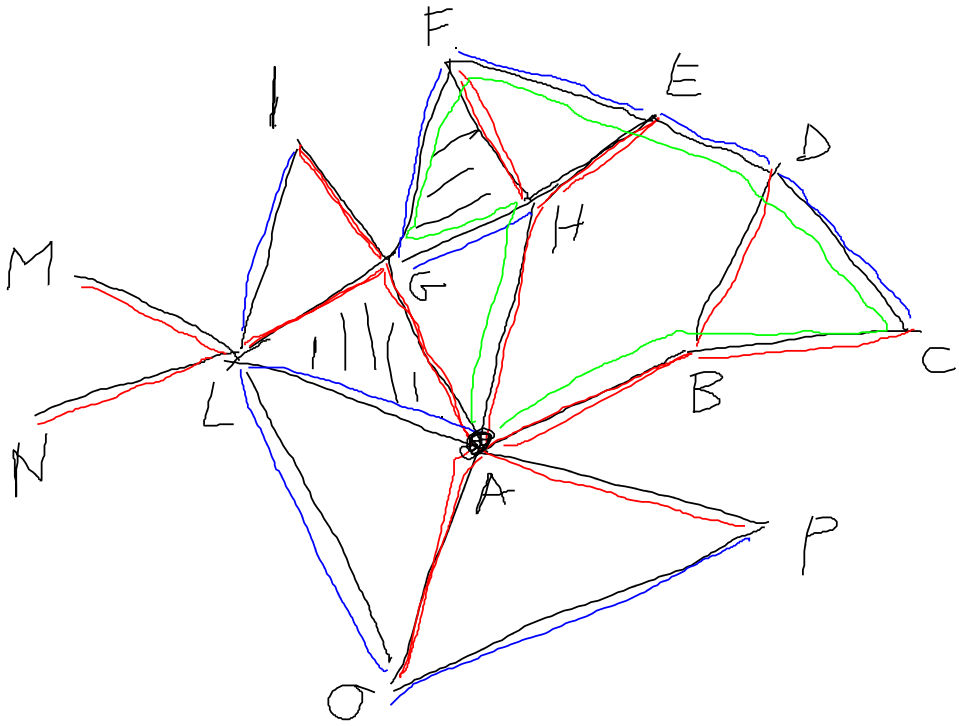


$A \mapsto A$

$B \mapsto AB$

$C \mapsto ABC$

$D \mapsto ABD$

$H \mapsto AH$

$E \mapsto AHE$

$F \mapsto AHF$

$G \mapsto AG$

$I \mapsto AGI$

$L \mapsto AL$

$\ldots$

A B C D E F G H A

$AB \mapsto ABA$ $\quad BC \mapsto ABBCBA$

$CD \mapsto ABCDBA$

$DE \mapsto ABDEHA$

$EF \mapsto AHEFHA$

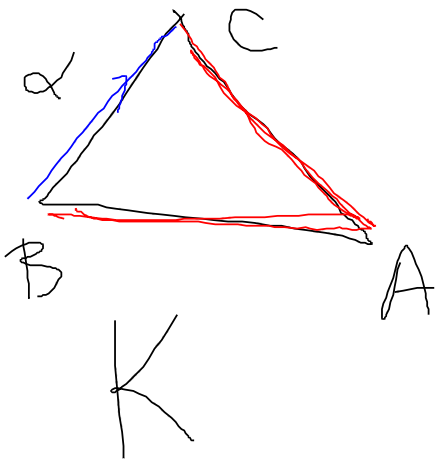
$FG \mapsto AHFGA$

$GH \mapsto AGHA$

$HA \mapsto AHA$

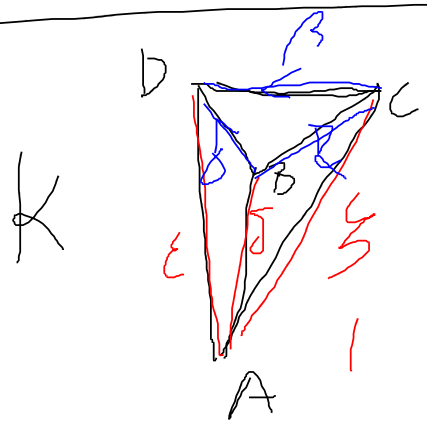
ABAABBCBAABCBBADEHAAHEFHAAHFGA

$\hookrightarrow$ AGHAAHA = ABCDEFGHA



$$E(K,A) \cong \langle \alpha \mid \rangle$$

$$\cong \mathbb{Z}$$



$$E(K,A) =$$

$$= \langle \alpha, \beta, \gamma \mid 1\cdot\alpha = 1,\ 1\cdot\beta = 1,\ 1\cdot\gamma = 1 \rangle$$

$1\cdot\alpha = 1$: ABC; $1\cdot\beta = 1$: ACD; $1\cdot\gamma = 1$: ABD

$$= 0$$

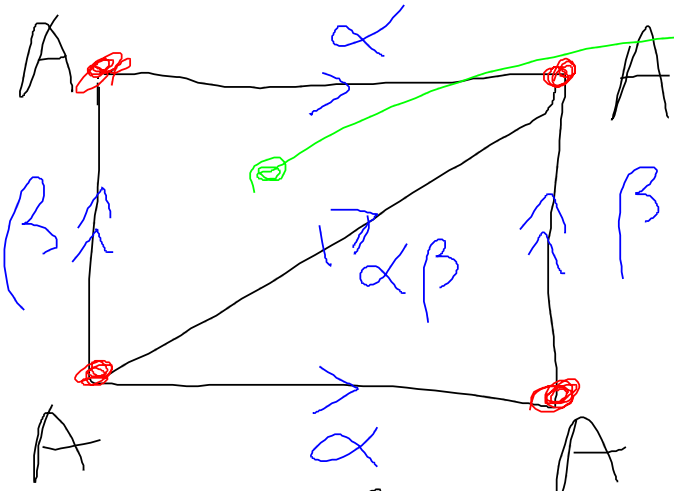
$$E(K,A) = \langle \alpha, \beta, \gamma, \delta, \varepsilon, \zeta \mid$$

$$\mid \delta, \varepsilon, \zeta,\ \delta\alpha = \zeta,\ \zeta\beta = \varepsilon,\ \delta\gamma = \varepsilon \rangle =$$

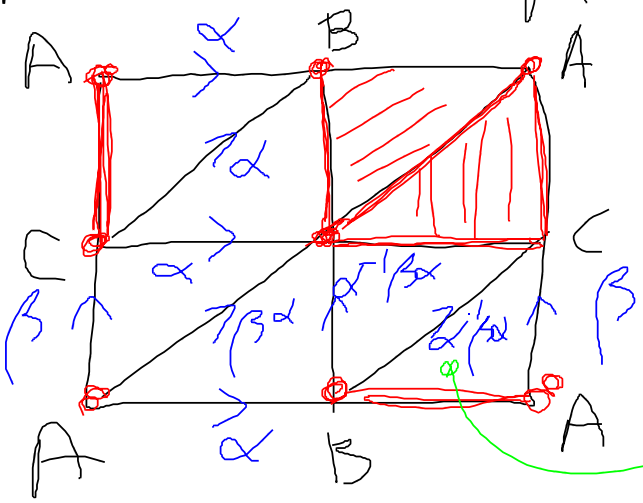
$$= \langle \alpha, \beta, \gamma \mid \alpha, \beta, \gamma \rangle = 0$$

$$\pi_1(T) \cong \langle \alpha, \beta \mid \alpha\beta\alpha^{-1}\beta^{-1} \rangle$$

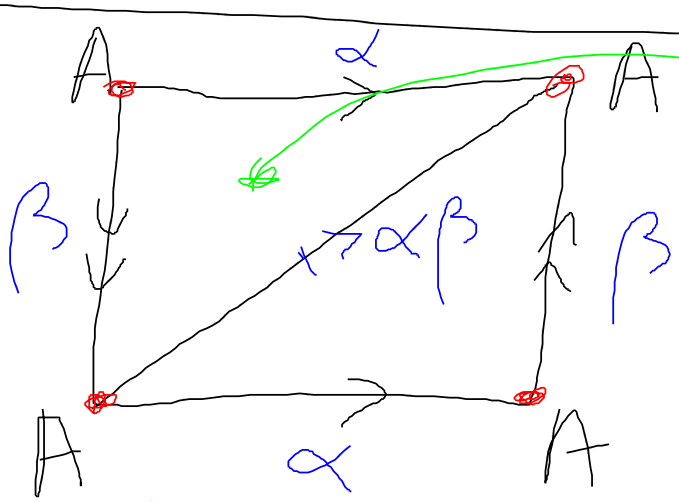
$$\cong \mathbb{Z} \oplus \mathbb{Z}$$



$$\alpha\beta = \beta\alpha$$
$$\alpha\beta\alpha^{-1}\beta^{-1} = 1$$



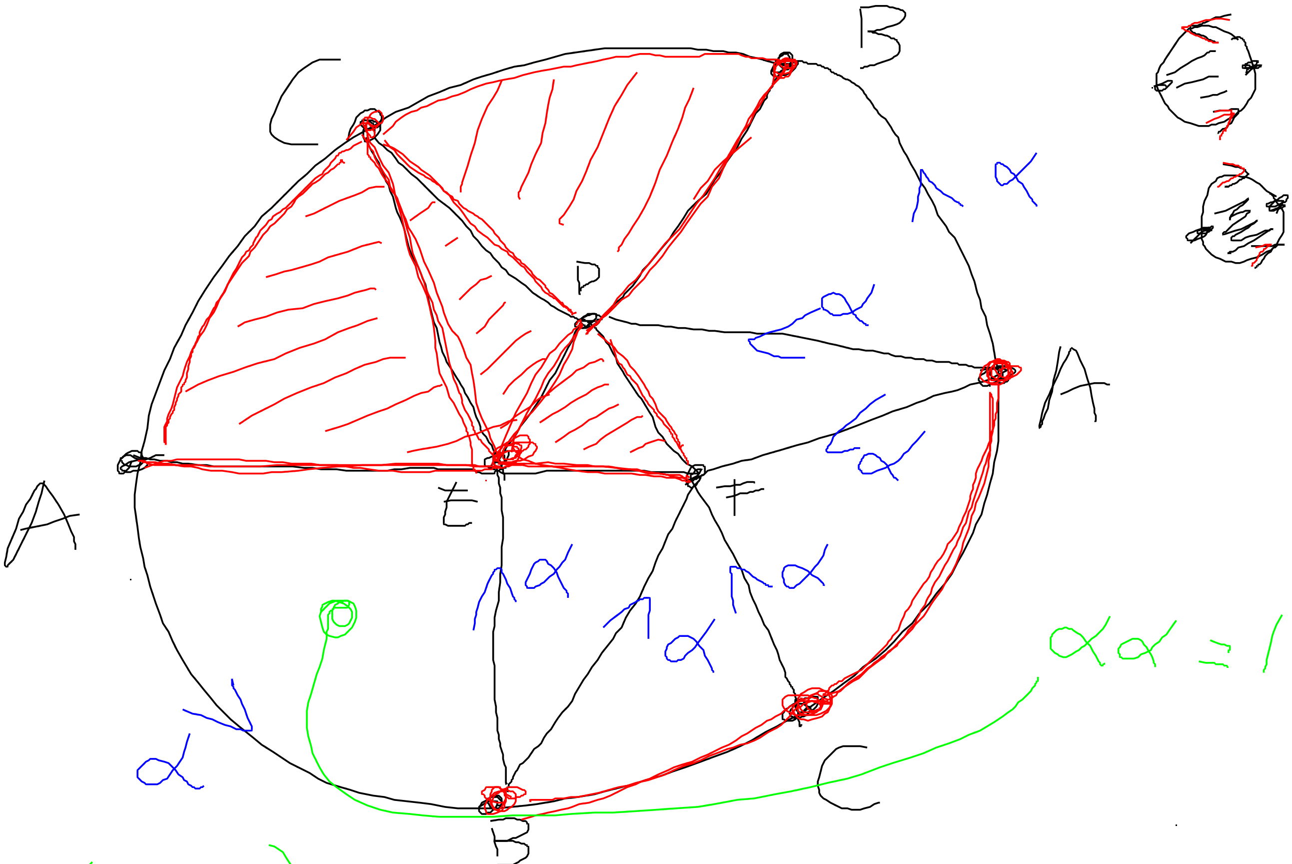
$$\alpha^{-1}\beta\alpha\beta^{-1} = 1$$
$$\alpha\beta\alpha = \beta$$
$$\beta^{-1}\alpha^{-1}\beta\alpha = 1$$



$$\alpha\beta\alpha^{-1}\beta = 1$$

$$\pi_1(\text{Klein}) \cong \langle \alpha, \beta \mid \alpha\beta\alpha^{-1}\beta \rangle$$

U_2



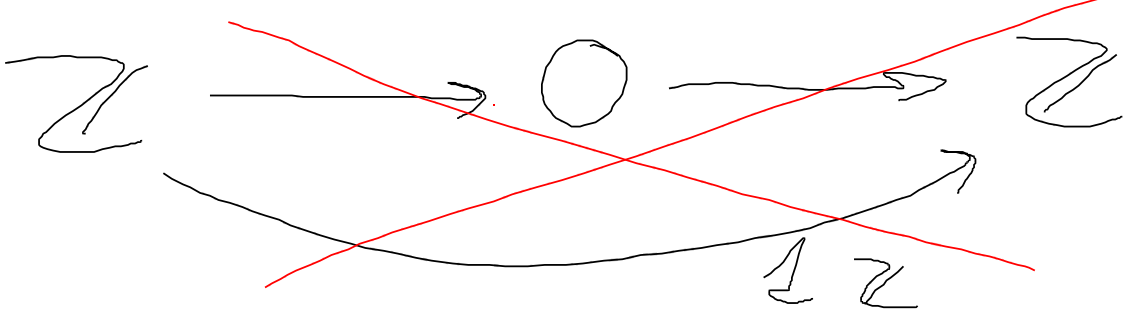
$$\pi_1(\mathbb{RP}^2) \cong \langle \alpha \mid \alpha^2 \rangle \cong \mathbb{Z}_2$$

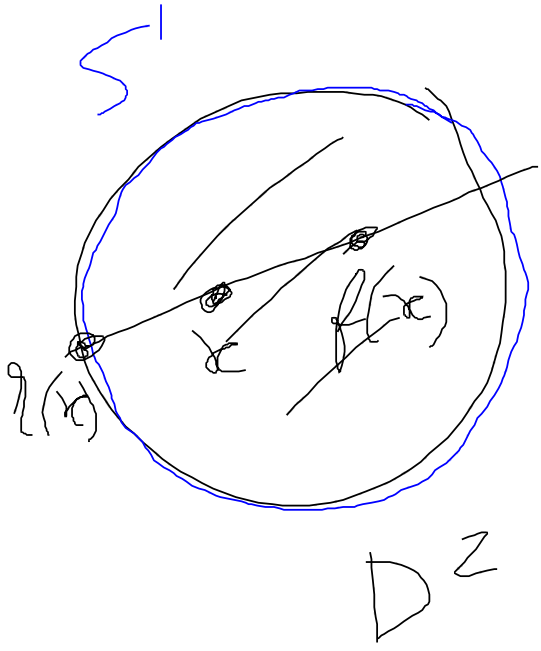
$$S^1 \xrightarrow{\ i\ } D^2 \xrightarrow{\ r\ } S^1$$

(composite: 1_{S^1})

$$\pi_1(S^1, x_0) \xrightarrow{\ i_\#\ } \pi_1(D^2, x_0) \xrightarrow{\ r_\#\ } \pi_1(S^1, x_0)$$

(composite: $1_{\pi_1(S^1, x_0)}$)


Z
Z
1 Z


S¹
D²