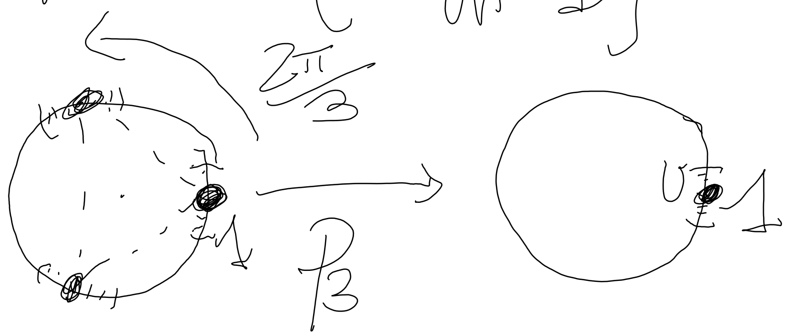
lunedì 20 aprile 2020 08:51

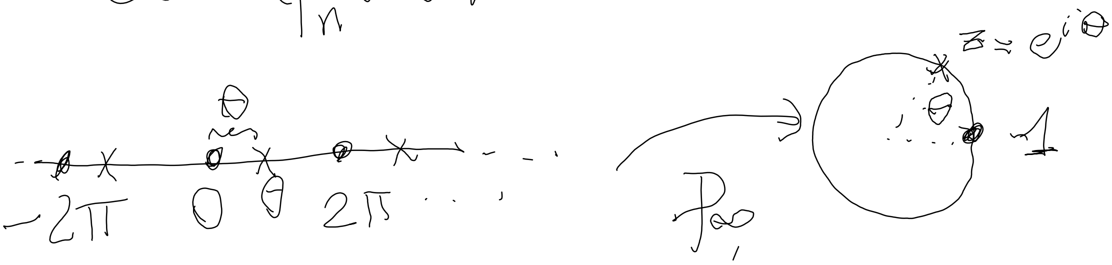
$$p_n : S^1 \longrightarrow S^1 \qquad n \in \mathbb{N}$$
$$z \longmapsto z^n$$

$p_n^{-1}(1) = \{$ radici n-esime di 1 $\}$

$n = 3$



$$\mathrm{card}(p_n^{-1}(z)) = n$$

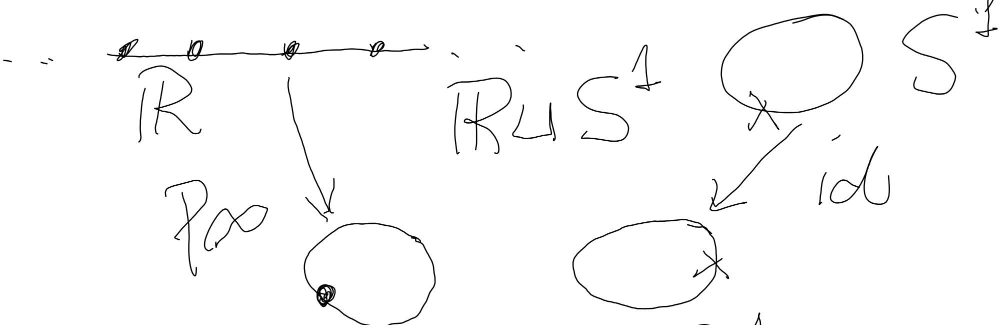


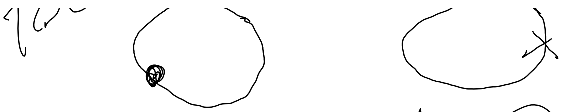
$$x \longrightarrow e^{ix}$$

$$p^{-1}(1) = 2\pi\mathbb{Z} = \{2k\pi : k \in \mathbb{Z}\}$$

$$p : \tilde{X} \longrightarrow X$$

$$\mathrm{card}(p^{-1}(x)) = \text{cost} \quad \forall x \in X \;?$$





$X = S^1 \sqcup S^1$

card$(p^{-1}(x))$ e' localmente costante

X connessa $\Longrightarrow$ costante

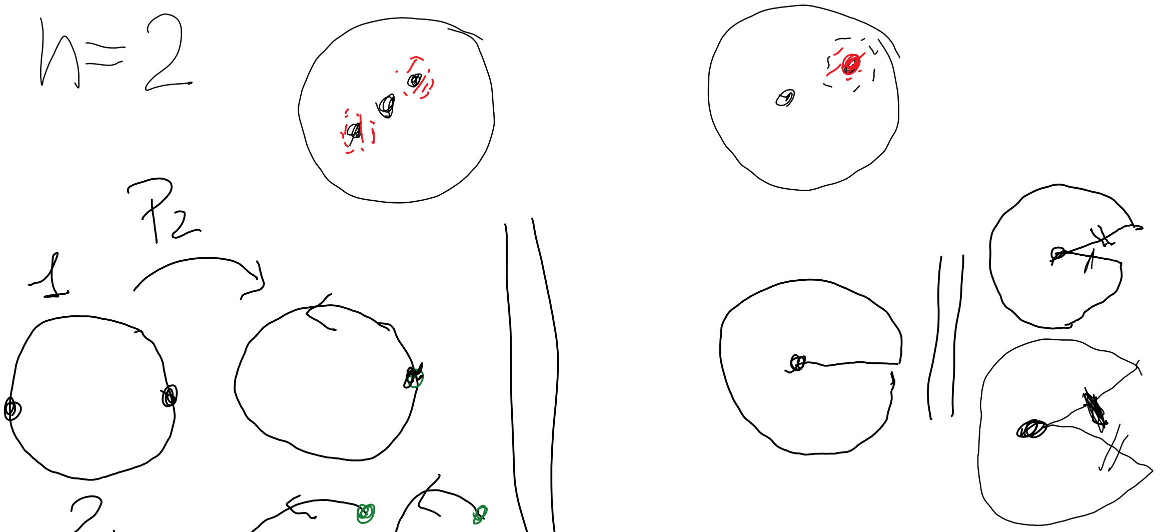
$D^2 \longrightarrow D^2 \quad n > 1$

$z \longrightarrow z^n$

$p^{-1}(0) = \{0\} \qquad |p^{-1}(z)| = n$ ($z \neq 0$)

NON RIVEST

$n = 2$



RIVESTIMENTO RAMIFICATO

$I \times D^2 \xrightarrow{\tilde{p}_n}$

$(t, z) \longrightarrow (t, z^n)$

$(t,z) \longrightarrow (t,z^n)$

$\mathbb{R}^3\setminus\{0\} \supseteq S^2 \xrightarrow[2:1]{} \mathbb{RP}^2$ $\qquad n:1$

$(x,y,z) \longmapsto [(x,y,z)]$

Rivestim. a 2 fogli o doppio

$\mathbb{C}^2\setminus\{0\} \longrightarrow \mathbb{CP}^1$

$(z,w)\sim(z',w')$ $\exists \lambda\in\mathbb{C}$ $\lambda(z,w)=(z',w')$ $\lambda\neq 0$

$(z,w) \longmapsto [z,w]$

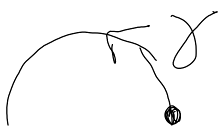
$$\{(z,w)\in\mathbb{C}^2 : \underbrace{\|(z,w)\|=1}_{z=a+ib \quad w=x+iy}\} \cong S^3$$

$S^3 \longrightarrow \mathbb{CP}^1$

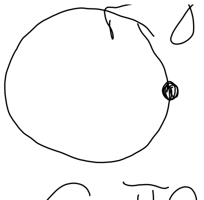
È UN RIVESTIMENTO?

GUARDARE LA FIBRA!

- RESTRIZ. DI RIVEST È RIVEST?
- COMPOZ. RIVESTIV. È RIVEST.?



$\pi_1(S^1, \cdot) = \langle\gamma\rangle \cong \mathbb{Z},+$



$\pi_1(S^1, \cdot) = \langle \gamma \rangle \cong \mathbb{Z},+$

SOTTOGRUPPI di $\mathbb{Z}$

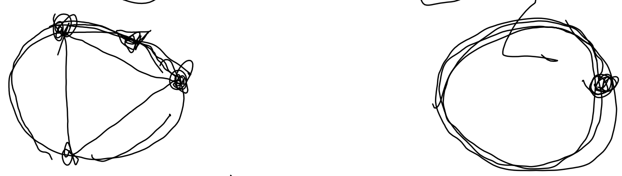
$\langle n \rangle$
$\langle 1 \rangle$
$\langle 0 \rangle$

$$p_\infty : \mathbb{R} \longrightarrow S^1$$

$$(p_\infty)_\# : \pi_1(\mathbb{R}) \longrightarrow \pi_1(S^1)$$

$\operatorname{Im}((p_\infty)_\#)$ = sottogruppo banale

$$p_n : S^1 \longrightarrow S^1$$

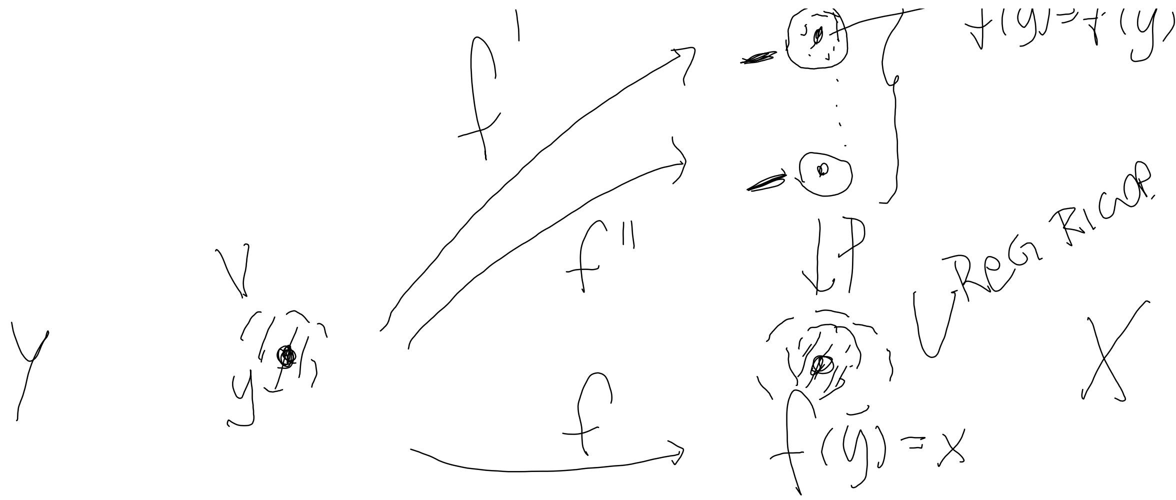


$$\operatorname{Im}((p_n)_\#) = \langle \gamma^n \rangle$$

$$\begin{tikzcd}
 & (\tilde{X}, \tilde{x}_0) \arrow[d, "p"] \\
(Y, y_0) \arrow[ur, dashed, "f'"] \arrow[r, "f"'] & (X, x_0)
\end{tikzcd}$$

$$p f' = f$$

n'  $f(y) = f'(y)$

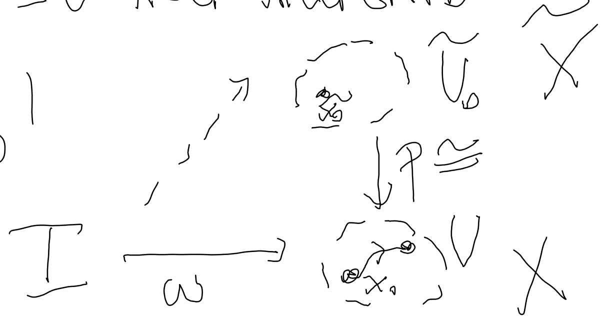


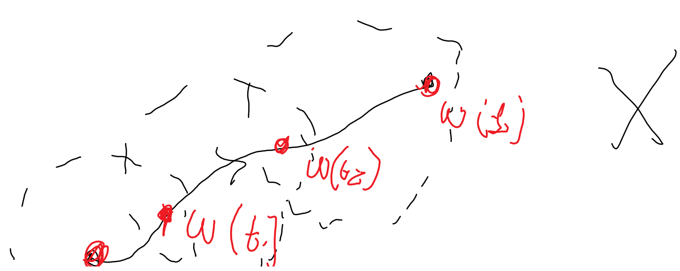
TH 6.3 $\quad Y = \{y_0\} \Longrightarrow$ TH 6.2

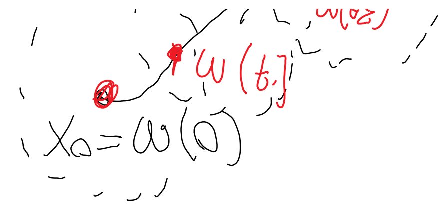
DIM TH 6.2.

IPOTESI $\quad \omega(I) \subseteq U$ REG RICOPERTO

$$(p|_{\tilde{U}_0})^{-1} \circ \omega = \omega'$$







$\forall x \in \omega(I)\ \exists\, U_x \ni x$ REG RICOP.

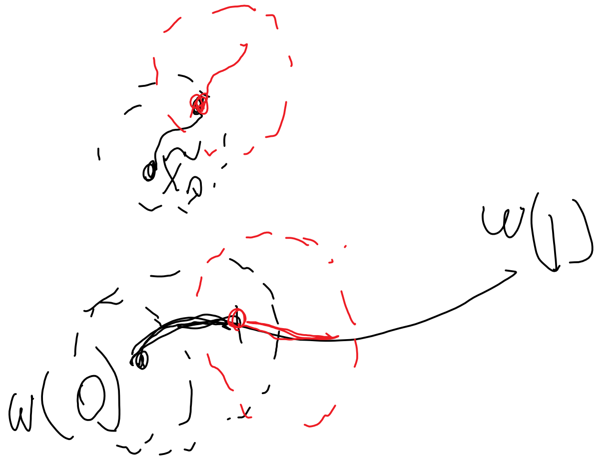
$\{\omega^{-1}(U_x)\}_{x \in \omega(I)}$ RICOPR. APERTO di I

I compattezza $\Rightarrow \exists$ sottoR. COPR. FINITO

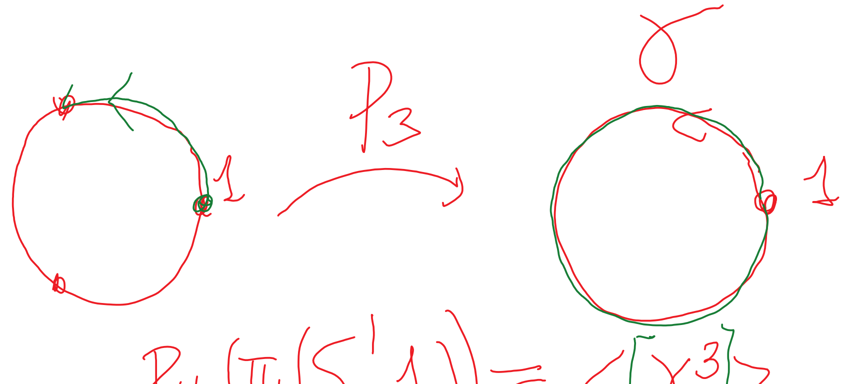
ora $0 = t_0 < t_1 < \dots < t_n = 1$

$$\omega([t_i, t_{i+1}]) \subseteq U_{x_i}$$

$$t_{i+1} \in U_{x_i} \cap \underbrace{U_{x_{i+1}}}_{X}$$



X



$$p_{3\#}(\pi_1(S^1, 1)) = \langle [\gamma^3] \rangle$$

$$\gamma_{3\#}(\pi_1(S^1,1)) = \langle [\gamma^3] \rangle$$

$$\mathbb{R}^2 \longrightarrow \mathbb{R} \times S^1$$
$$(h, x) \longmapsto (h, e^{ix})$$