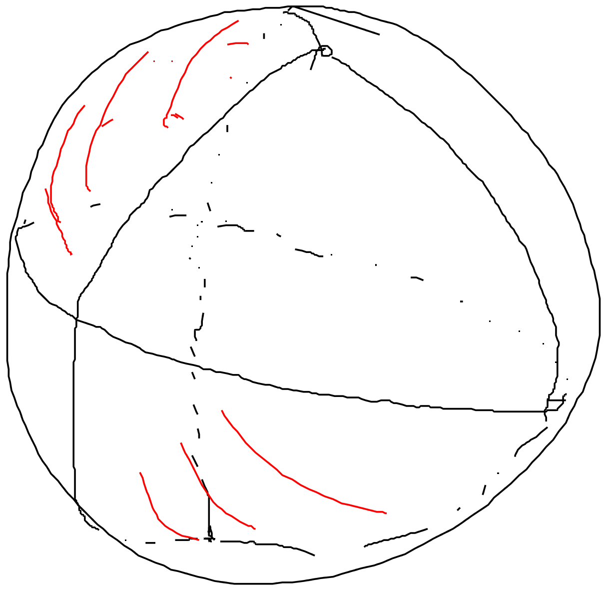
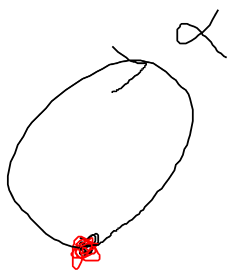
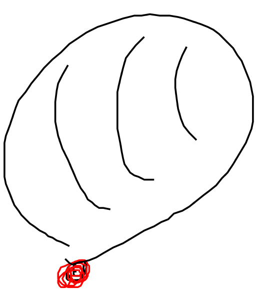
$L(3,1)$

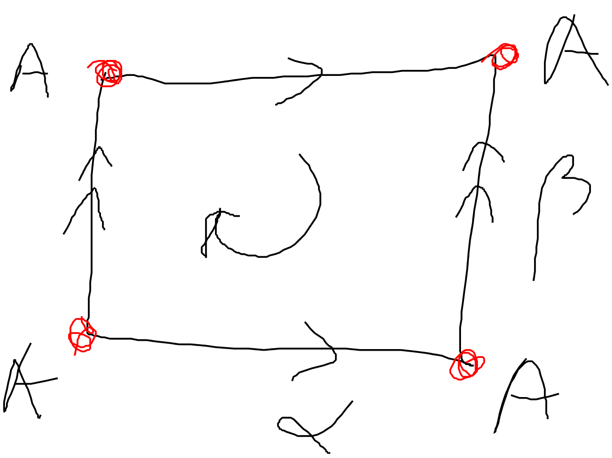




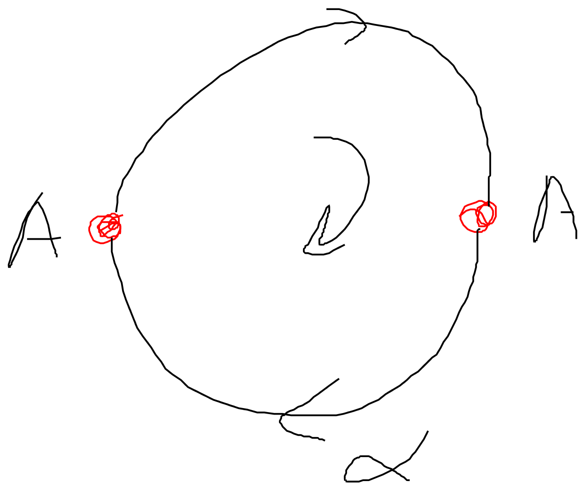
$\langle \alpha \mid \ \rangle$



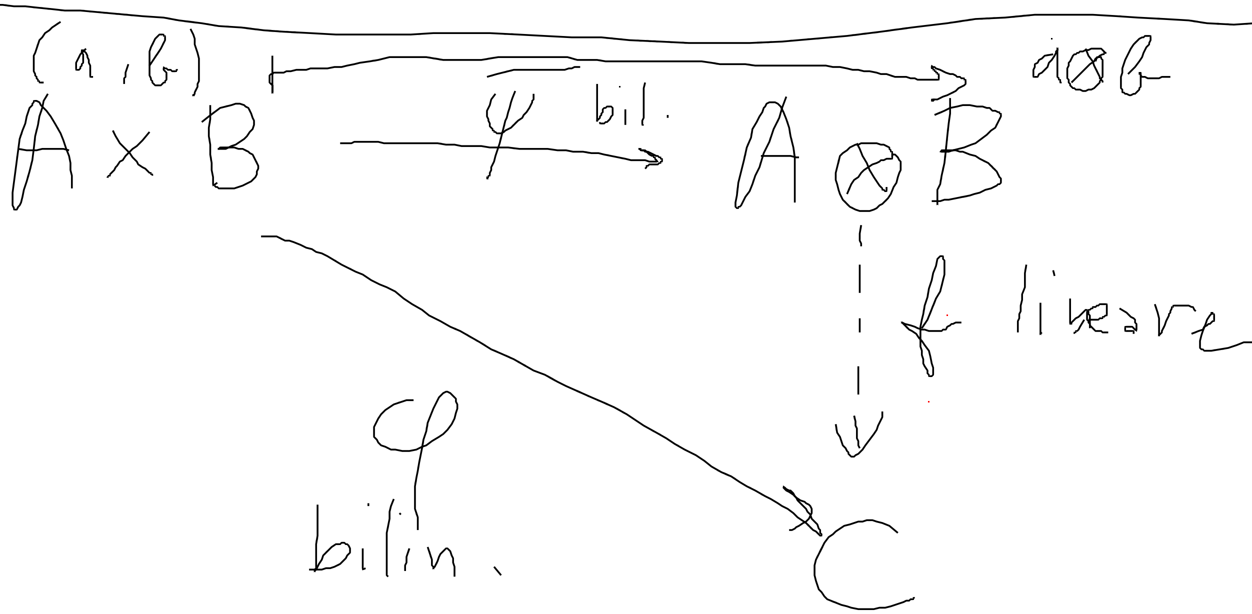
$\langle \ \mid \ \rangle$



$\langle \alpha, \beta \mid \alpha\beta^{-1}\alpha^{-1}\beta \rangle$



$\langle \alpha \mid \alpha^2 \rangle$



$\mathbb{Z} \otimes \mathbb{Z}$

elementi di $\mathbb{Z} \otimes \mathbb{Z}$ sono combi lin. di elementi

$m \otimes n$

$$m \otimes n = m(1 \otimes n) = m \cdot n \cdot (1 \otimes 1)$$

Ma allora effettivamente ogni elemento di $\mathbb{Z} \otimes \mathbb{Z}$ è una comb. lineare (a scalari in $\mathbb{Z}$) del solo $1 \otimes 1$.

Perciò $\mathbb{Z} \otimes \mathbb{Z} \cong \mathbb{Z}$

$\mathbb{Z} \otimes \mathbb{Z}_3 \qquad \mathbb{Z}_3 = \{\bar{0}, \bar{1}, \bar{2}\}$

Gli elementi di $\mathbb{Z} \otimes \mathbb{Z}_3$ sono combin. lin. (a coeff. in $\mathbb{Z}$) di elementi

$m \otimes \bar{n}$

Ma

$m \otimes \bar{n} = m(1 \otimes \bar{n}) = (1 \otimes \overline{(mn)})$

$m \otimes \bar{0} = (1 \otimes \overline{(m \cdot 0)}) = 1 \otimes \bar{0} = 0 \cdot (1 \otimes \bar{1})$

$m \otimes \bar{1} = (1 \otimes \overline{(m \cdot 1)}) = m \cdot (1 \otimes \bar{1})$

$m \otimes \bar{2} = (1 \otimes \overline{(2m)}) = 2m \cdot (1 \otimes \bar{1})$

Perciò sono tutte e sole le comb. lineari di $1 \otimes \bar{1}$

Dunque $\mathbb{Z} \otimes \mathbb{Z}_3 \cong \mathbb{Z}_3$

$\mathbb{Z}_2 \otimes \mathbb{Z}_3$ $\qquad$ $\mathbb{Z}_2 = \{\tilde{0}, \tilde{1}\}$

Abbiamo comb. lin. (a coeff. in $\mathbb{Z}$) di elementi $\tilde{m} \otimes \bar{n}$

$\tilde{1} \otimes \bar{1}$

$$\tilde{1} \otimes \bar{2} = \tilde{1} \otimes (2\bar{1}) = 2(\tilde{1} \otimes \bar{1}) = (\tilde{2} \otimes \bar{1}) = \tilde{0} \otimes \bar{1} = \tilde{0} \otimes \bar{0}$$

$$\tilde{1} \otimes \bar{0} = \tilde{1} \otimes (0 \cdot \bar{1}) = (0 \cdot \tilde{1} \otimes \bar{1}) = \tilde{0} \otimes \bar{0}$$

$$\tilde{0} \otimes \bar{n} = (\tilde{1} \otimes 0 \cdot \bar{n}) = \tilde{1} \otimes \bar{0} = \tilde{0} \otimes \bar{0}$$

$$\tilde{1} \otimes \bar{1} = \tilde{1} \otimes 4\bar{1} = 4(\tilde{1} \otimes \bar{1}) = (4\tilde{1} \otimes \bar{1}) = \tilde{0} \otimes \bar{1} = \tilde{0} \otimes \bar{0}$$

$\mathbb{Z}_2 \otimes \mathbb{Z}_3$ è il gruppo banale

$\mathbb{Z}_2 \otimes \mathbb{Z}_6$ $\qquad$ $\mathbb{Z}_6 = \{\bar{\bar{0}}, \dots, \bar{\bar{5}}\}$

$$\tilde{1} \otimes \bar{\bar{1}} = \tilde{1} \otimes (7\bar{\bar{1}}) = (7\tilde{1}) \otimes \bar{\bar{1}} = \tilde{1} \otimes \bar{\bar{1}}$$

$$\tilde{1} \otimes \bar{\bar{3}} = \tilde{1} \otimes (3\bar{\bar{1}}) = (3\tilde{1}) \otimes \bar{\bar{1}} = \tilde{1} \otimes \bar{\bar{1}}$$

$$\tilde{1} \otimes \bar{\bar{4}} = \tilde{1} \otimes (4\bar{\bar{1}}) = (4\tilde{1}) \otimes \bar{\bar{1}} = \tilde{0} \otimes \bar{\bar{1}} = (0 \cdot \tilde{0}) \otimes \bar{\bar{1}} = \tilde{0} \otimes (0\bar{\bar{1}}) = \tilde{0} \otimes \bar{\bar{0}}$$

$\mathbb{Z}_2 \otimes \mathbb{Z}_6$ contiene solo $\tilde{0} \otimes \bar{\bar{0}}$ e $\tilde{1} \otimes \bar{\bar{1}}$

$\cong$

$\mathbb{Z}_2$

PROP

$$\mathbb{Z} \otimes \mathbb{Z} \simeq \mathbb{Z}$$

$$\mathbb{Z} \otimes \mathbb{Z}_p \simeq \mathbb{Z}_p$$

$$\mathbb{Z}_p \otimes \mathbb{Z}_q \simeq \mathbb{Z}_{MCD(p,q)}$$