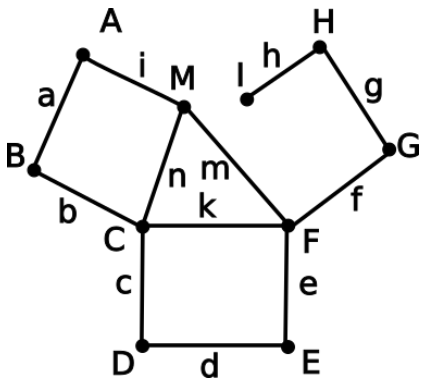
Mathematical Methods – 20 December 2024 – Graph Theory
UniBo matriculation number:
(no name, please)



Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F	G	H	I	M
A	0	1	0	0	0	0	0	0	0	1
B	1	0	1	0	0	0	0	0	0	0
C	0	1	0	1	0	1	0	0	0	1
D	0	0	1	0	1	0	0	0	0	0
E	0	0	0	1	0	1	0	0	0	0
F	0	0	1	0	1	0	1	0	0	1
G	0	0	0	0	0	1	0	1	0	0
H	0	0	0	0	0	0	1	0	1	0
I	0	0	0	0	0	0	0	1	0	0
M	1	0	1	0	0	1	0	0	0	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i	k	m	n
A	1	0	0	0	0	0	0	0	1	0	0	0
B	1	1	0	0	0	0	0	0	0	0	0	0
C	0	1	1	0	0	0	0	0	0	1	0	1
D	0	0	1	1	0	0	0	0	0	0	0	0
E	0	0	0	1	1	0	0	0	0	0	0	0
F	0	0	0	0	1	1	0	0	0	1	1	0
G	0	0	0	0	0	1	1	0	0	0	0	0
H	0	0	0	0	0	0	1	1	0	0	0	0
I	0	0	0	0	0	0	0	1	0	0	0	0
M	0	0	0	0	0	0	0	0	1	0	1	1

3) *(1 pt.)* Minimum degree δ = 1 Maximum degree Δ = 4

4) *(1 pt.)* Connectivity κ = 1 Edge-connectivity κ' = 1

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No. It contains odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No. It contains vertices of odd degree.

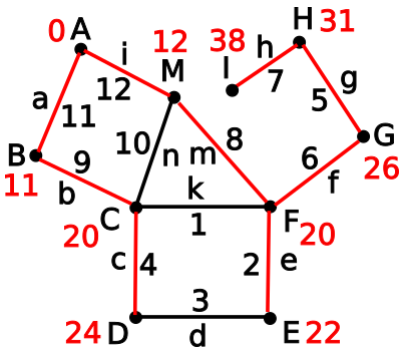
7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)
Yes. It contains only two vertices of odd degree. hgfedcbainkm

8) *(1 pt.)* Does G have a Hamilton cycle? (If answer is "yes", write the vertex sequence of one)
No.

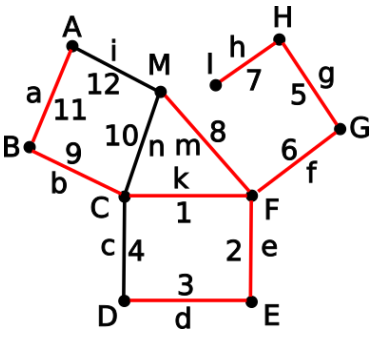
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
{a,n,d,f,h} Yes.

Now the vertices represent towns, and the edge weights represent distances.

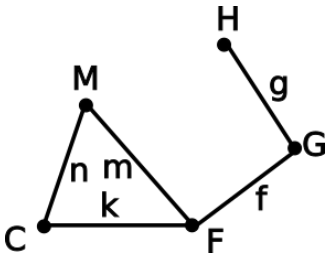
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



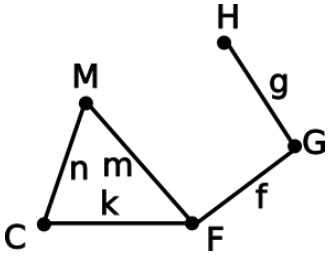
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 3



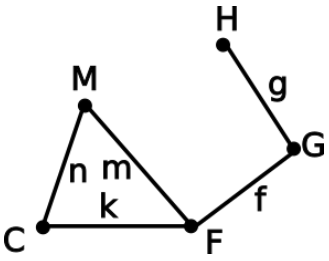
13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).



(M+CF)(C+MF)(F+CMG)(G+FH)(H+G) =
= (MC+MMF+CFC+CFMF)( ) = (MC+MF+CF)(F+CMG)( ) =
= (MCF+MCCMG+MFF+MFCMG+CFF+CFCMG)( ) =
= (CMG+MFG+MFH+CFG+CFH)(H+G) =
= CMGH+CMGG+MFGH+MFGG+MFHH+MFHG+CFGH+CFGG+CFHH+CFHG =
= CMG+MFG+MFH+CFG+CFH

Minimal coverings: {C,M,G}, {M,F,G}, {M,F,H}, {C,F,G}, {C,F,H}
Maximal independent sets: {F,H}, {C,H}, {C,G}, {M,H}, {M,G}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$k^5-5k^4+9k^3-7k^2+2k$