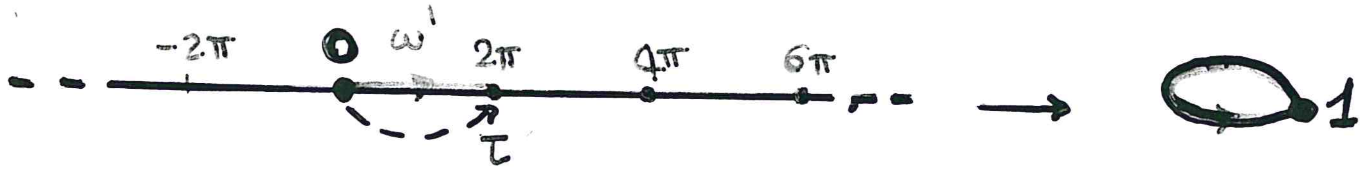
$p: (\mathbb{R}, 0) \longrightarrow (S^1, 1)$

$t \longmapsto e^{it}$



$\Phi \in Aut(p)$

$2\pi\mathbb{Z} = p^{-1}(\{1\}) = \Phi(0)$

$[p(\omega')] \in \pi_1(S^1, 1)$

$\omega'(0) = 0 \quad \omega'(1) = \Phi(0)$

$2\pi \longleftrightarrow \tau$ traslazione di un vettore $\longleftrightarrow \partial$:

generatore di $\pi_1(S^1, 1)$

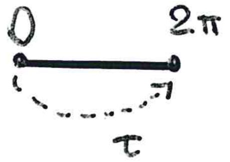
$\underbrace{0 \longrightarrow}_{2\pi}$

$\tau : \mathbb{R} \longrightarrow \mathbb{R}$

$\tau(t) = t + 2\pi$

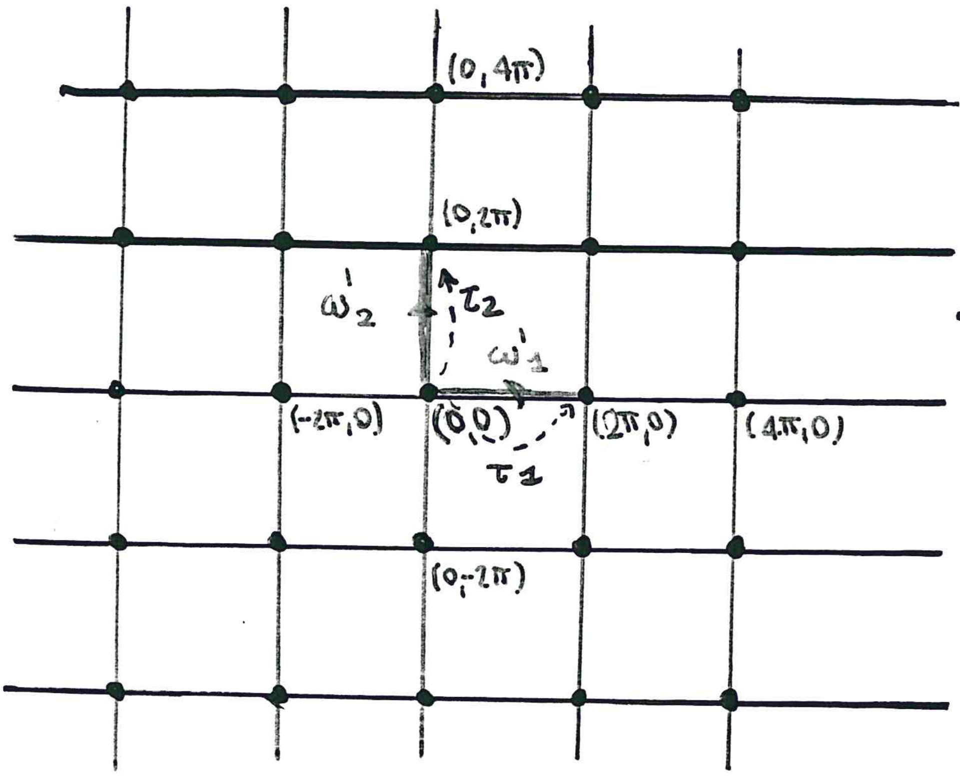
$$Aut\ p = \langle \tau \rangle \cong \mathbb{Z}$$

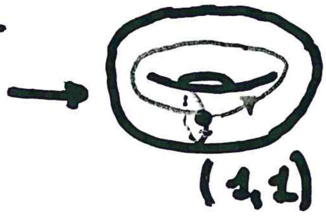
$$[0, 2\pi]/\sim \; = \mathbb{R}/Aut(p) \cong S^1$$



$\tau(0) = 2\pi$

$$p : (\mathbb{R}\times\mathbb{R}, (0,0)) \longrightarrow (S^1\times S^1, (1,1)) = (T, (1,1))$$
$$(t,s) \longmapsto (e^{it}, e^{is})$$





FIBRA di $(1,1)$
$\{(2\pi k, 2\pi h)\ h,k \in \mathbb{Z}$

$\pi_1(T,(1,1)) = \langle a\rangle \oplus \langle b\rangle$

- $(2\pi, 0) \longleftrightarrow \tau_1$ traslazione di un vettore $\longleftrightarrow$ a:

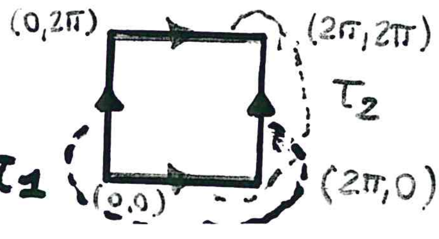
$$\tau_1 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$(t,s) \longmapsto (t+2\pi, s)$$

- $(0, 2\pi) \longleftrightarrow \tau_2$ traslazione di un vettore $\longleftrightarrow$ b:

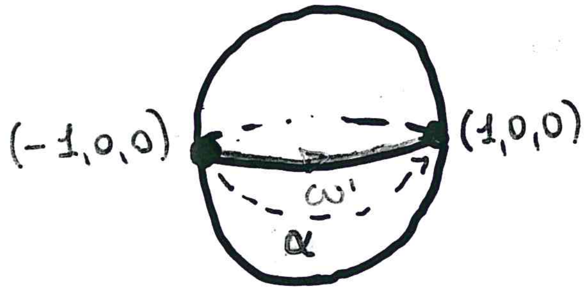
$$\tau_2 : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$(t,s) \longmapsto (t, s+2\pi)$$

$$\mathrm{Aut}\, p = \langle \tau_1 \rangle \oplus \langle \tau_2 \rangle \cong \mathbb{Z} \oplus \mathbb{Z}$$

$$\mathbb{R}^2 / \mathrm{Aut}(p) = \frac{[0,2\pi]\times[0,2\pi]}{\sim} \cong T$$



$$p : (S^2, (1,0,0)) \longrightarrow (\mathbb{RP}^2, [1,0,0])$$
$$(x,y,z) \longmapsto [x,y,z]$$



$$\longrightarrow \mathbb{RP}^2$$

$$(-1,0,0) \longleftrightarrow \alpha : S^2 \to S^2 \longleftrightarrow a = [p(\omega')]$$
$$(x,y,z) \mapsto (-x,-y,-z)$$

FIBRA DI $[1,0,0]$
È $\{(1,0,0),(-1,0,0)\}$

$$\mathrm{Aut}(p) = \langle \alpha \rangle \cong \mathbb{Z}_2 \cong \pi_1(\mathbb{RP}^2, [1,0,0])$$

$$S^2/_{\mathrm{Aut}\,p} = D^2/_{\sim} \cong \mathbb{RP}^2$$