# GLOBAL FUNDAMENTAL SOLUTIONS FOR HÖRMANDER OPERATORS VIA A SATURATION METHOD

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In an article of 1977, Folland built a global solution for a Hörmander differential operator of order 2, $\mathcal{L} = \sum X_i^2$ on $\mathbb{R}^m$, where the $X_i$s are homogeneous vector fields. Posing the homogeneity hypothesis, it is in fact possible to globalize the lifting and approximation approach of the paramount works of Rothschild-Stein and Goodman. The main idea by Folland, was to consider the Carnot group $\mathbb{G}$ generated by the $X_i$s, and see $\mathcal{L}$ as the projection of the sub-Laplacian $\mathcal{L}_{\mathbb{G}}$, which is known to have a fundamental solution. In a series of recent works, Biagi and Bonfiglioli generalized this approach to any differential operator $\mathcal{L}$ over $\mathbb{R}^m$ generated by homogeneous vector fields. They represented the Carnot group as a direct product $\mathbb{R}^n \times \mathbb{R}^m$ and proved that any fundamental solution of the lifted differential operator, can be "saturated" to obtain a fundamental solution $\Gamma$ for $\mathcal{L}$.

I present a similar result that applies to any Riemannian manifold and vector fields respecting the Hörmander condition. In particular, it is possible to represent the group $G$ generated by the $X_i$s as a direct product with the fibers having a group structure. This allows to expand the saturation approach of Biagi-Bonfiglioli and therefore to project-down a vast class of fundamental solutions.