Boundary Harnack principles and Schauder estimates for degenerate equations on singular sets

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joint works with
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The ratio v/u of two solutions to a second order elliptic equation in divergence form solves a degenerate elliptic equation if u and v share the zero set; that is, $Z(u) \subseteq Z(v)$. The coefficients of the degenerate equation vanish on the nodal set as u^2. Developing a Schauder theory for such equations, we prove $C^{k,\alpha}$-regularity of the ratio from one side of the regular part of the nodal set in the spirit of the higher order boundary Harnack principle established by De Silva and Savin in [4]. Then, by a gluing lemma, the estimates extend across the regular part of the nodal set. Eventually, using conformal mapping in dimension $n = 2$, we provide local gradient estimates for the ratio which hold also across the singular part of the nodal set and depends on the highest value attained by the Almgren frequency function.

References

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