Regularity of the optimal sets for spectral functionals and the free boundary for the vectorial Bernoulli problem

Susanna Terracini

Università di Torino

`susanna.terracini@unito.it`

In this talk we deal with the regularity of optimal sets for a shape optimization problem involving a combination of eigenvalues, under a fixed volume constraints. As a model problem, consider

$$\min\left\{\lambda_1(\Omega) + \cdots + \lambda_k(\Omega) \ : \ \Omega \subset \mathbb{R}^d, \text{ open }, \ |\Omega| = 1\right\},$$

where $\lambda_i(\cdot)$ denotes the eigenvalues of the Dirichlet Laplacian and $|\cdot|$ the d-dimensional Lebesgue measure. We prove that any minimizer Ω_{opt} has a regular part of the topological boundary which is relatively open and $C^{1,\alpha}$ regular and that the singular part has Hausdorff dimension smaller than $d - d^*$, where $d^* \geq 3$ is the minimal dimension allowing the existence of minimal conic solutions to the bow-up problem.

We examine the link between this and the problem of regularity of the free boundary for a vector-valued Bernoulli problem, with no sign assumptions on the boundary data. More precisely, given an open, smooth set of finite measure $D \subset \mathbb{R}^d$, $\Lambda > 0$ and $\varphi_i \in H^{1/2}(\partial D)$, we deal with

$$\min\left\{\sum_{i=1}^{k} \int_D |\nabla v_i|^2 + \Lambda \left| \bigcup_{i=1}^{k} \{v_i \neq 0\}\right| \ : \ v_i = \varphi_i \text{ on } \partial D\right\}.$$

We prove that, for any optimal vector $U = (u_1, \ldots, u_k)$, the free boundary $\partial(\cup_{i=1}^{k}\{u_i \neq 0\}) \cap D$ is made of a regular part, which is relatively open and locally the graph of a C^∞ function, a (one-phase) singular part, of Hausdorff dimension at most $d - d^*$, for a $d^* \in \{5, 6, 7\}$, and by a set of branching (two-phase) points, which is relatively closed and of finite $\mathcal{H}^{d-1}$ measure having a stratified structure itself.

Thess are joint works with Dario Mazzoleni and Bozhidar Velichkov.

References

[1] D. Mazzoleni, S. Terracini and B. Velichkov, Regularity of the optimal sets for spectral functionals, Geom. Funct. Anal. 27 (2017), no. 2, 373-426

[2] Dario Mazzoleni, Susanna Terracini, Bozhidar Velichkov, Regularity of the free boundary for the vectorial Bernoulli problem, preprint 2018