New Results on Instantaneous Blowup
in $\mathbb{H}^N$

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Consider the heat equation

$$\frac{\partial u}{\partial t} = \Delta u + V(x)u$$

for $x \in \mathbb{R}^N$ with a positive potential $V(x)$. If V is "too singular", then this equation may not have any positive solutions, as was discovered in 1984. We shall discuss the history of the problem as well as later developments, including new results obtained in 2017. The Euclidean space $\mathbb{R}^N$ can be replaced by the Heisenberg group $\mathbb{H}^N$ and other Carnot groups, and the heat equation can be relaced by the Ornstein-Uhlenbeck equation and other related equations. Some nonlinear results will be mentioned. Scaling plays a critical role.