Classification of fused links

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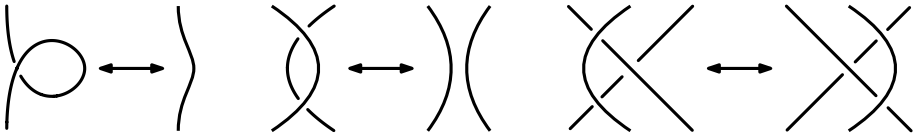
Classical knots and links can be described in various ways. Given a method of description, however, there may be more than one description that represents the same link. For example, a common method of describing a link is a planar diagram called a link diagram. An n-component link is represented as generic immersion of n disjoint circles in the plane (link diagram) where double points have an additional information on overpasses and underpasses. 1-component link is called knot.

Classical crossings

Link diagrams are equivalent under ambient isotopy and some types of local moves (Reidemeister moves).

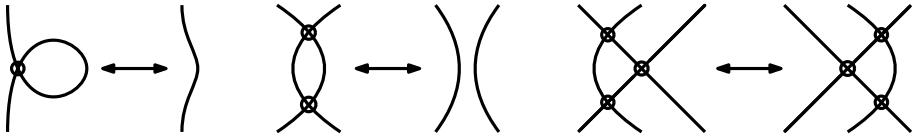


Classical Reidemeister moves

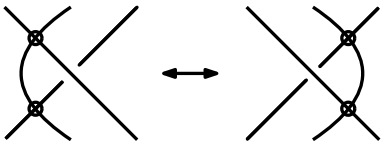
Virtual link diagram is a generalization of classical link diagram: this is represented as generic immersion of circles in the plane, but each vertex is now allowed to be a classical crossing or a new type called virtual.

Virtual crossing

Virtual link diagrams are equivalent under ambient isotopy, classical Reidemeister moves, virtual Reidemeister moves and Mixed Reidemeister moves.

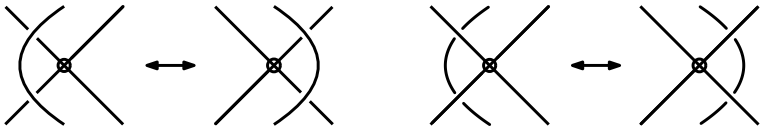


Virtual Reidemeister moves



Mixed Reidemeister moves

Classical knot theory is a subtheory of Virtual knot theory in sense that two classical links, which are equivalent as virtual links are also equivalent as classical links. Note that the following local moves are forbidden for virtual links.



Forbidden moves

Fused link diagram is a simplification of virtual link diagram which allows forbidden moves. In the theory of fused links every knot is equivalent to the trivial knot. However not every link is equivalent to the trivial link. For example, trivial 2-component link, Hopf link and Hopf link with one virtual crossing and with one classical crossing all are different.

The full classification of fused links is not (completely) trivial. In particular, A. Fish and E. Keyman proved that the fused link with classical crossings only is completely determined by the linking numbers of each pair of components.

We find all non-equivalent classes of fused links and construct an easy computable complete invariant for fused links valued in the infinitely generated free abelian group. We prove that the set of equivalence classes of n-component fused links is in one-to-one correspondence with the set of elements of the abelization UVP_n/UVP'_n up to conjugation by the elements from the symmetric group $S_n < UVB_n$.