Coercivity of integral functionals of non-everywhere superlinear lagrangians

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Abstract

We consider the following non-autonomous variational problem:

minimize
$$\left\{ F(v) = \int_{a}^{b} f(x, v(x), v'(x)) \, \mathrm{d}x \, : \, v \in \Omega \right\}$$

where $\Omega := \{ v \in W^{1,1}(a,b), v(a) = A, v(b) = B, v(x) \in I \}.$

The Lagrangian F is assumed to have just a "non-everywhere" superlinear growth, being allowed to vanish at some $x_0 \in [a, b]$, or $s_0 \in I$. We prove some sufficient conditions ensuring the coercivity of the functional F. As a consequence, when f is convex with respect to the last variable, the existence of the minimum can be immediately derived.