NONLOCAL FRACTIONAL PROBLEMS VIA VARIATIONAL METHODS

Fractional and nonlocal operators appear in concrete applications in many fields such as, among the others, optimization, finance, phase transitions, stratified materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science and water waves. This is one of the reason why, recently, nonlocal fractional problems are widely studied in the literature.

Aim of this talk will be to present some recent results for nonlocal problems modeled by

$$(0.1) \qquad \begin{cases} (-\Delta)^s u - \lambda u = f(u) & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^n \setminus \Omega \,, \end{cases}$$

where $s \in (0,1)$ is fixed, $\Omega \subset \mathbb{R}^n$, $n > 2s$, is an open bounded set with continuous boundary, λ is a real parameter, $(-\Delta)^s$ is the fractional Laplace operator , which (up to normalization factors) may be defined as

$$-(-\Delta)^s u(x) = \int_{\mathbb{R}^n} \frac{u(x+y) + u(x-y) - 2u(x)}{|y|^{n+2s}} \, dy \,, \quad x \in \mathbb{R}^n \,,$$

while f is a nonlinear term satisfying suitable growth assumptions. In problem (0.1) the Dirichlet datum is given in $\mathbb{R}^n \setminus \Omega$ and not simply on $\partial \Omega$, consistently with the nonlocal character of the operator $(-\Delta)^s$.

These results were obtained through variational and topological methods and extend the validity of some theorems known in the classical case of the Laplacian to the nonlocal fractional framework.