INFINITE DIMENSIONAL STOCHASTIC CALCULUS, AN INFINITE DIMENSIONAL PDE AND SOME APPLICATIONS

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Abstract This talk develops some aspects of stochastic calculus via regularization for processes with values in a general Banach space B. A new concept of quadratic variation which depends on a particular subspace is introduced. An Itô formula and stability results for processes admitting this kind of quadratic variation are presented. Particular interest is devoted to the case when B is the space of real continuous functions defined on $[-T, 0]$, $T > 0$ and the process is the window process $X(\cdot)$ associated with a continuous real process X which, at time t, it takes into account the past of the process. If X is a finite quadratic variation process (for instance Dirichlet, weak Dirichlet), it is possible to represent a large class of path-dependent random variable h as a real number plus a real forward integral in a semiexplicite form. This representation result of h makes use of a functional solving an infinite dimensional partial differential equation. This decomposition generalizes, in some cases, the Clark-Ocone formula which is true when X is the standard Brownian motion W. Some stability results will be given explicitly.
This is a joint work with Francesco Russo (ENSTA ParisTech Paris).

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