Singular analysis of the optimizers of the principal eigenvalue in weighted Neumann problems

Dario Mazzoleni

Università di Pavia

Abstract

We study the minimization of the positive principal eigenvalue associated to a weighted Neumann problem settled in a bounded smooth domain $\Omega \subset \mathbb{R}^N$, within a suitable class of sign-changing weights. This problem naturally arises in population dynamics. Denoting with u the optimal eigenfunction and with D its super-level set associated to the optimal weight, we perform the analysis of the singular limit of the optimal eigenvalue as the measure of D tends to zero. We show that, when the measure of D is sufficiently small, u has a unique local maximum point lying on the boundary of Ω and D is connected. Furthermore, the boundary of D intersects the boundary of the box Ω, and more precisely, $\mathcal{H}^{N-1}(\partial D \cap \partial\Omega) \geq C|D|^{(N-1)/N}$ for some universal constant $C > 0$. Though widely expected, these properties are still unknown if the measure of D is arbitrary.

This is a joint project with B. Pellacci and G. Verzini.