

LIPSCHITZ REGULARITY OF ALMOST MINIMIZERS FOR THE p -LAPLACIAN

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In this talk, we deal with almost minimizers in one-phase problems driven by the p -Laplace operator. More specifically, we study nonnegative almost minimizers for the energy functional

$$(1) \quad J_p(u, \Omega) := \int_{\Omega} \left(|\nabla u(x)|^p + \chi_{\{u>0\}}(x) \right) dx, \quad p > 1,$$

where Ω is a bounded domain in \mathbb{R}^n . The functional J_p is a generalization to each $p > 1$ of the classical one-phase (Bernoulli) energy functional, which corresponds to the choice of $p = 2$ in (1). We denote then by J_2 such functional.

Almost minimizers of J_2 were investigated recently in [2, 1]. However, in [4] D. De Silva and O. Savin provided a different approach than [2, 1], based on nonvariational techniques, to deal with almost minimizers of J_2 and their free boundaries. Precisely, inspired by their work [5], they showed that almost minimizers of J_2 are “viscosity solutions” in a more general sense. Once this was established, the regularity of the free boundary for almost minimizers followed by employing the techniques developed by De Silva in [3].

In our talk, we present a main Lipschitz continuity result about almost minimizers of J_p , $p > \max \left\{ \frac{2n}{n+2}, 1 \right\}$. Our approach is inspired by the method introduced in [4]. In particular, we discuss the main steps of this approach, focusing on the main ideas. This talk is based on a joint work with S. Dipierro, F. Ferrari and E. Valdinoci, see [6].

REFERENCES

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