Recent results on evolution equations of Black-Scholes and Cox-Ingersoll-Ross type and their associated semigroups: analogies and differences

Silvia Romanelli
Università degli Studi di Bari Aldo Moro
silvia.romanelli@uniba.it

Abstract

Consider the following initial value problem

$$\frac{\partial u}{\partial t} = \nu^2 x^k \frac{\partial^2 u}{\partial x^2} + (\gamma + \beta x)\frac{\partial u}{\partial x} + (cx + d)u, \ t \geq 0, x \geq 0, \tag{1}$$

$$u(x,0) = f(x), \ x \geq 0, \tag{2}$$

where $\nu > 0, \gamma \geq 0$, $\beta \in \mathbb{R}$ and $c, d \leq 0$. We focus on the cases when (1) is either the Black-Scholes equation (BS), i.e., $k = 2, \quad \gamma = 0 = c, \quad \beta > 0, \quad d = -\beta$, or a Cox-Ingersoll-Ross-type equation (CIR), i.e., $k = 1, \quad \gamma > 0, \quad \beta \in \mathbb{R}, \quad c < 0, \quad d = 0$.
In both cases we will present existence and regularity results for the (C_0) semigroups associated to (1)-(2) in suitable spaces of continuous functions on $(0, \infty)$ highlighting, in particular, analogies and differences between the corresponding generators.
Note that, if $\sqrt{2}\nu$ is the volatility of the stochastic interest rate, under suitable assumptions about the financial markets, (BS) models the pricing of a European call option while (CIR) models the pricing of a discount zero coupon bond.