THE MINSKY MOMENT AS THE REVENGE OF ENTROPY

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Abstract: Considering macroeconomies as systems subject to stochastic forms of entropic equilibria, we shall consider how deviations driven by positive feedbacks as in a speculative bubble can drive such an economy into an anti-entropic state that can suddenly collapse back into an entropic state, with such a collapse taking the form of a Minsky moment. This can manifest itself as shifts in the boundary between the portion of the income distribution that is best modeled as Boltzmann-Gibbs and that best modeled as a Paretian power law.

Introduction

The idea of viewing the economy as an entropic process has a long and controversial history, dating arguably back to the energetics movement of the turn of the last century (Ostwald, 1908). Julius Davidson (1919) would argue that the foundation of the law of diminishing returns was the Second Law of Thermodynamics. Alfred J. Lotka (1925), the father of the idea of predator-prey cycles, would argue that evolution is a grand entropic process that encompasses the economy within its dynamics. Paul Samuelson (1947) would draw heavily on the categories of dynamical states proposed by Lotka, but abjured the idea that the law of entropy was relevant to economics, sneering at those who invoke it in his Nobel Prize address as being "crackpots" (Samuelson, 1972). Nevertheless, along with drawing on the ideas of Lotka in his 1947 *Foundations of Economic Analysis*, he also ultimately drew on the statistical mechanics ideas of J. Willard Gibbs (1902), with statistical mechanics the main area of application of the law of entropy. Eventually Samuelson (1990, p. 263) admitted that, "It is the *mathematical* structure of *classical* (phenomenological, macroscopic, nonstochastic) *thermodynamics* that has isomorphisms with *theoretical economics*," even though the idea of a nonstochastic thermodynamics is rather empty.

With the rise of *econophysics* the use of the idea of entropy in economics has re-emerged (Rosser, 2016). We can distinguish those that are ontological from those that are metaphorical. The former rely on the idea that the economy itself is fundamentally embodied as a part of the broader physical and ecological system, with the entropic processes operating through the laws of thermodynamics determining the passage of energy from the sun through the biosphere (Georgescu-Roegen, 1971) as well as driving the forward process of evolution as one of a process of unfolding and emerging complexity (Lotka, 1925). In this view energy is the fundamental resource and entropy is the fundamental reality of the economy.

In the metaphorical perspective it is more the mathematics of entropy that matters as a way of thinking about dynamical economic systems, with this in a sense more in line with the Samuelson quotation above from 1990, even as the more recent adopters of the entropy idea have pushed beyond uses that Samuelson made or might have approved of. These have included among others alternative stochastic definitions of general economic equilibria (Föllmer, 1974; Foley, 1994), financial modeling (Cozzolino and Zahner, 1973; Stutzer, 1994, 2000), and income and wealth distribution dynamics (Bouchaud and Mézard, 2000; Chakraborti and Chakraborti, 2000; Dragulescu and Yakovenko, 2001; Solomon and Richmond, 2002).

Considering the application of the entropy concept to financial modeling and in conjunction with the idea of an alternative entropic formulation of economic equilibria suggests a possible application to financial-economic modeling, in particular to conceptions of financial dynamics along the lines suggested by Minsky (1972, 1982). In this view, the stages of psychological transition in financial markets as confidence expands during a boom period and lending restrictions relax essentially involves the emergence of a positive feedback anti-entropic process that creates an ultimately unsustainable order in the form of the rising speculative bubbles that come out of such processes. This leads to the idea that the end of such a process in the form of a collapse of the bubbles in a “Minsky moment” represents the workings of the law of entropy and a return to the stochastic entropic equilibrium state, a “revenge of entropy,” if you will.

This financial market outcome can then generate an outcome in terms of income distribution dynamics in an economy. It is well established (Clementi and Gallegati, 2005; Yakovenko and Rosser, 2009) that income distribution can be well modeled by breaking the distribution into two portions. One covering the lower portion, around 98 percent of the population give or take some, is modeled best by a Boltzmann-Gibbs distribution. This is consistent with arguments long made that wage and salary dynamics tend to follow lognormal processes over time as individuals randomly move up or down following various individual shocks. However the top layer of the income distribution exhibits a stronger influence of capital income as well as salaries more closely related to capital income, with these tending to move with financial market returns dynamics. These are well established to tend to follow power law dynamics, and wealth distributions also tend to reflect such a pattern.

Thus when markets are sharply rising in a speculative bubble, reflecting an anti-entropic mechanism, the proportion of income going to capital income earners tends to rise, with this tending to push the boundary between the two groups of income distribution downwards to occupy a larger portion of the distribution. However, when the bubble crashes and a more entropic pattern results, the share of capital income declines and the boundary should shift back upwards to reflect a greater share going to labor income. Hence a Minsky moment can become the revenge of entropy.

Defining Entropy and its Forms in Economics

The most widely used form of the Boltzmann equation for entropy is on his grave, although he never wrote it down in that way (Uffink, 2014). It involves *W,* the thermodynamic probability of an aggregate state of a system of gas molecules, with *k* the Boltzmann constant, and *S* being entropy as

$$S = k \ln W. \qquad (1)$$

Given *N* microscopic states of the system, the probability of a gas molecule being in the ith state is N_i/N. *W* is then given by (Chakrabarti and Chakraborty, 2006)

$$W = N!/\Pi N_i!. \qquad (2)$$

This means that Boltzmann entropy can be rewritten as

$$S = k \ln (N!/\Pi N_i!). \qquad (3)$$

Basic Shannon entropy is given by *H* of the probability distribution of states of informational uncertainty for message i. of $H(p_1 \ldots p_n)$. This then equals (Shannon and Weaver, 1949; Rényi, 1961)

$$H(p_1 \ldots p_n) = -k \,\Sigma\, p_i \ln p_i. \qquad (4)$$

Recognizing that $p_i = N_i/N$, the basic unity of these two concepts appear as *N* increases, which leads the Boltzmann formula in (3) to approach (Tsallis, 1988; Thurner and Hanel,2012)

$$S = -kN\Sigma\, p_i \ln p_i, \qquad (5)$$

which means that in the limit as *N* approaches infinity, Boltzmann entropy is proportional to Shannon entropy.

Ontological and Metaphorical Entropy

The ontological approach to econophysics derives from the direct and foundational role of energy in the economy, not merely for industrial production or providing for electricity or transportation (Cockshott, et al., 2009), but at the ecological or biophysical level, that of solar energy driving the global biosphere. This is more a return to the Carnot (1824) and Clausius (1885), who studied the dynamics of steam engines, view of thermodynamics, where the continued incoming of solar energy shows the openness of the earth's system that allows it to avoid the law of entropy as long as the sun lasts (Georgescu-Roegen, 1971; Daly, 1987). However, that arriving solar energy itself is finite and thus provides a direct limit on economic activity that depends on the ecosystems through which the solar energy dissipates in the food chains that are driven by that energy. In addition, Georgescu-Roegen (1971) extended this argument to broader material resource inputs, arguing that they are also subject to a form of the law of entropy as well that provides further limits on the economy. More broadly for him [1971, p. 281] "the economic process consists of a continuous transformation of low entropy into high entropy, that is, into *irrevocable waste*, or, with a topical term, into pollution."

While variations of this argument have become highly influential, especially in ecological economics as with Martinez-Alier (1987), it has faced sharp criticisms as well. Thus, Gerelli (1985) argues that the scale of the solar input is such that it is orders of magnitude beyond really limiting the world economy, with many other more mundane constraints more relevant in the short run. Nordhaus (1992) estimated entropy to be as many as 12 orders of magnitude below technology as a limit to growth, with Young (1994) weighing in similarly. In that regard the drawdown of stored energy sources and their limits such as with fossil fuels may be more relevant with the pollution from using them even more limiting as with such outcomes as climate change arising from the burning of such fuels releasing their stored carbon dioxide. Other critics have emphasized either the limitless ingenuity of the human mind such as Julian Simon, who argued that (1981, p. 347) "those who view the relevant universe as unbounded view the second law of thermodynamics as irrelevant to the discussion."

Another important figure in this line of argument was Alfred J. Lotka (1925), the father of the concept of predator-prey cycles. Lotka argued that the law of entropy is a deep driving force in evolution, a source of a teleological directedness of the process towards greater complexity. He saw this as the fundamental physical foundation of biology that needed to be studied mathematically, and he in turn saw the economy as deriving from the ecosystem as the more recent ecological economists have. Ironically Lotka was a tremendous influence on Paul Samuelson, who cited him prominently in his magnum opus, *Foundations of Economic Analysis* (1947), although more for his categorization of the stability conditions of linear systems rather than for his arguments regarding the law of entropy or its relation to the economy.

As already noted, Samuelson (1972) ridiculed many uses of the concept of entropy in economics as being "crackpot,"even as he recognized that the mathematics of entropy might be of some use in some circumstances. Applications of the law of entropy to energy flow systems as the foundation of economies, and especially those theories that have sought to make this the basis of a law of value within economics (Ostwald, 1908), have been among those ideas that drew these sorts of condemnations from Samuelson.

In contrast to this ontological perspective is a metaphorical one (Rosser, 2016) that is more in line with Samuelson's view of the appropriate way to use entropic ideas in economics. Here it is the mathematical formalism derived from Gibbsian statistical mechanics and applied in an appropriate way that is the key. This can be seen as the foundation for the Shannon approach to studying information problems, and it has been used in a variety of ways within economics. Among these are general equilibrium price determination, financial market dynamics, and income and wealth dynamics, as we shall see below.

Entropy and General Equilibrium Value

Moving to the heart of economics entropy has been proposed as an alternative to the conventional Arrow-Debreu explanation of value. That standard view has equilibrium being a vector of prices that are fixed points. The entropic alternative recognizes the reality of a stochastic world in which equilibrium is better depicted as a probability distribution of prices as prices are never the same everywhere at any point in time for any commodity except as measure zero accident. An early expression of this idea is due to Hans Föllmer (1974). A fuller development of this has been due to Foley (1994), later extended by Foley and Smith (2008).

The basic Foley (1994) model involves strong assumptions such as that all possible transactions within an economy have equal probability. However his solution involves a statistical distribution of behaviors in the economy where a particular transaction is inversely proportional to the exponential of its equilibrium entropy price, with this coming from a maximum Botlzmann-Gibbs entropy set of shadow prices. Walrasian general equilibrium is a special case of this model when "temperature" is zero. The more general form lacks the usual welfare implications, and it allows for the possibility of negative prices as in the case of Herodotus auctions (Baye et al., 2012).

Let there be *m* commodities, *n* agents of type *k* who achieve a transaction *x* of which there are is $h^k[x]$ proportion of agents type *k* out of *r* who do transaction *x* out of an offer set *A*, of which there are *mn*. *Multiplicity* of an assignment for *n* agents assigned to *S* actions, each of them *s*, is given by:

$$W[n_{s]}] = n!/(n_1!\ldots n_s!\ldots n_S!). \qquad (6)$$

Shannon entropy of this multiplicity is given by:

$$H\{h^k[x]\} = -\Sigma_{k=1}{}^r W^k \Sigma_{x\epsilon Ak} h^k[x]x = 0. \qquad (7)$$

Maximizing this entropic formulation subject to the appropriate feasibility constraints, which if non-empty, gives the unique canonical Gibbs solution:

$$H^k[x] = exp[-\Pi x]/\Sigma_x exp[-\Pi x], \quad (8))$$

where Π are vectors of the entropy shadow prices.

Entropic Financial Modeling

Central to all financial analysis is concern with how to model price, risk, and uncertainty. This inevitably involves study of probability distributions and stochastic processes. Unsurprisingly statistical physics has provided models and inspiration for doing this, including at times the concept of entropy from Boltzmann and Gibbs are drawn on. However, whether the thermodynamical processes are driving the economy through industrial production or through foundational biophysical systems it is the mathematics of Shannon and other entropies that can be used to understand these stochastic processes in a metaphorical fashion.

In this regard the analogy to Shannon entropy, which some of these models draw on specifically, it can be argued that Shannon entropy itself is a metaphor in a way that Boltzmann-Gibbs entropy is not, or especially that Carnot-Clausius entropy is not. Again, the latter is an ontological foundation of physical phenomena, the dissipation of heat energy in mechanical processes initially, but then, inspired by Maxwell, providing a mathematics to explain heat itself, Shannon entropy deals with something more abstract, information, although that certainly has its real world uses, as does financial modeling. However, it is harder to say that the law of entropy itself is what is driving these phenomena rather than that the mathematics of entropy is useful for understanding or explaining them.

On the title page of his *Foundations of Economic Analysis* [30], Paul Samuelson famously quoted Gibbs as saying, “Mathematics is a language.” That it certainly is. But in the case of Shannon entropy, as well as financial models based on entropy mathematics, it is a metaphor rather than a linguistic ontology.

Drawing on much discussion from various econophysicists, Schinkus (2009) argues that econophysicists are more inclined than regular economists to approach data without preconceptions regarding distributions or parameter values, although they may be more inclined to draw on ideas from physics, with entropy among those in connection with financial modeling. Thus, Dionisio et al (2009, p. 161) argue that:

> “Entropy is a measure of dispersion, uncertainty, disorder and diversification used in dynamic process, in statistics and information theory, and has been increasingly adopted in financial theory.”

Applications of the law of entropy using Shannon entropy or Boltzmann-Gibbs distributions easily fit into explaining or modeling distributions that rely on lognormality, which are easily consistent with Gaussian approaches. While we know that ultimately these entropies are essentially identical mathematically, the real difference is that one we believe is driven to maximization as a law of physics whereas in the more metaphorical ones observing an extremum for entropy is simply a useful mathematical condition.

Someone drawing on both of the main measures of entropy in order to develop core financial theory in the form of the Black-Scholes options pricing formula (1973) is Michael J. Stutzer (1994, 2000). In the second of these he used Shannon entropy for his generalization of the link, after pointing out that Cozzolino and Zahner (1973) had used Shannon entropy to derive lognormal stock price distributions, the same year that Black and Scholes (1973) published their result without directly relying on any entropy mathematics. For his generalization Stutzer (2000) posed the problem in discrete form as considering a stock market price process given by

$$\Delta p/p = \mu\Delta t + \sigma\sqrt{\Delta t}\Delta z, \quad (9)$$

where p is price, t is the time interval, and the second term on the right hand side is the random shock, with these distributed $\sim N(0, \Delta t)$. With Q as quantity, $r\Delta t$ the riskless rate of return, and P the actual conditional risk density distribution, a central focus is the conditional risk neutral given by dQ/dP.

From these one considers the relative entropy minimizing conditional risk neutral density that in effect maximizes order

$$arg\ min_{dQ/dP} \int log\ dQ/dP\ dQ, \quad (10)$$

subject to a martingale restriction given by

$$r\Delta t - E[(\Delta p/p)(dQ/dP] = 0, \quad (11)$$

From this he shows that when asset returns are IID with normally distributed shocks as given above, the martingale product density formed from the relative entropy minimizing conditional risk is that used to calculate the Black-Scholes option pricing formula. He recognizes that this does not easily generalize to non-Gaussian distributions such as the power law ones much studied by econophysicists, suggesting a weaker approach using Generalized Auto Regressive Conditional Heteroskedastic (GARCH) processes.

The Anti-Entropic Minsky Bubble Process

As discussed above, entropy maximization implies Gaussian stochastic dynamics. These are not consistent with power law distributions seen in financial market returns or in wealth distributions. A likely source for this difference is the tendency to anti-entropic bubble dynamics that can be described by the Minsky process (Minsky, 1972; Kindleberger, 1978; Rosser, 1991). Rather than evening out irregularities, a speculative bubble can heighten deviations from long run equilibrium outcomes, whether of a stochastic entropic sort as modeled by Foley and others or a deterministic Walrasian general equilibrium. Positive feedback dynamics arising from momentum or noise traders drive prices to extremes away from these equilibria temporarily, generally ending with some sort of crash. These extreme movements lead to the kurtotic fat tails that appear in financial asset return dynamics so ubiquitously (Lux, 2009).

Minsky (1972) that these dynamics emerge endogenously through psychological mechanisms wherein agents become complacent regarding risk during periods of entropic equilibrium with Gaussian distributions predominating in response to exogenous shocks. They proceed through stages of increasing risk taking, wherein leverage ratios rise and bubbles emerge. The final stage of this process involves Ponzi dynamics that have become unhinged from fundamentals fully. Wealth has risen dramatically with the speculative bubble prices, but in the end the bubble crashes, usually in a dramatic Minsky Moment when panic takes over and agents sell the asset en masse (Kindleberger, 1972). With this the dynamic returns prices into the longer run entropic equilibrium zone, the “Revenge of Entropy.”

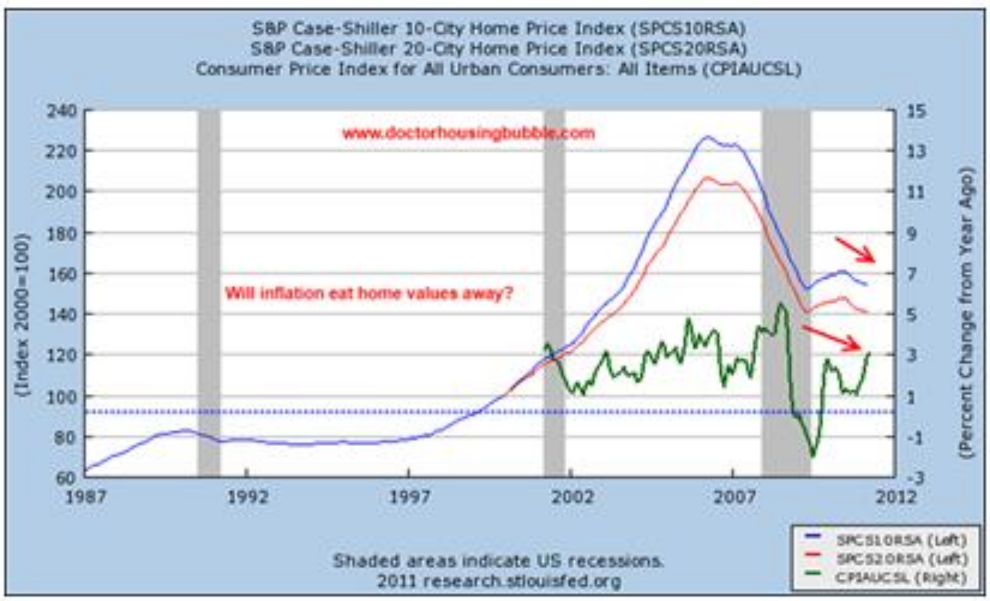
It has been well understood that such dynamics have historically generally taken one of three different forms (Rosser, 1991, Chap. 5). All three of these cases are shown in Figures 1-3 (Rosser, et al., 2012) for assets that exhibited each of them during the period of the Great Recession of 2007-2009, although the dynamics are shown for a longer time, with one of them (US housing) peaking prior to the broader financial crash that ushered in the Great Recession.

The first case involves a price that rises in an accelerating fashion, only to suddenly crash after peaking in a dramatic Minsky Moment. For the period of the financial crisis, this is well exhibited by oil prices, which peaked in July, 2008 at $147 per barrel only to decline sharply to around $30 per barrel in November, 2008. Such patterns are often seen in commodity price speculative bubbles.

Figure 1: Monthly West Texas Intermediate Crude Oil Prices per barrel, US $, 2003-2011

Source: Rosser, Rosser, and Gallegati (2012), Figure 1

The second case involves a more gradual rise in prices that then declines also in a gradual way after the peak is reached. Such a case can be argued not to possess a Minsky Moment proper in the sense of a sudden crash associated with a panic, although there may be panicky emotions involved for agents in such a bubble and its decline. The example from the financial crisis is that if US housing, whose prices began to rise in 1998 and then peaked in 2006, declining thereafter for several years as shown in Figure 2. Indeed real estate seems more prone to exhibit such a pattern, and one explanation that seems to hold especially for residential real estate is that people refuse to sell immediately in the downturn, believing that the prices are “unfair” and “too low.” Leading them to rent out their housing if they must move or simply refusing to sell. Such patterns thus tend to show a fall in volume of sales during the decline more than a rapid decline in price, which falls as eventually people give up and accept the lower prices.

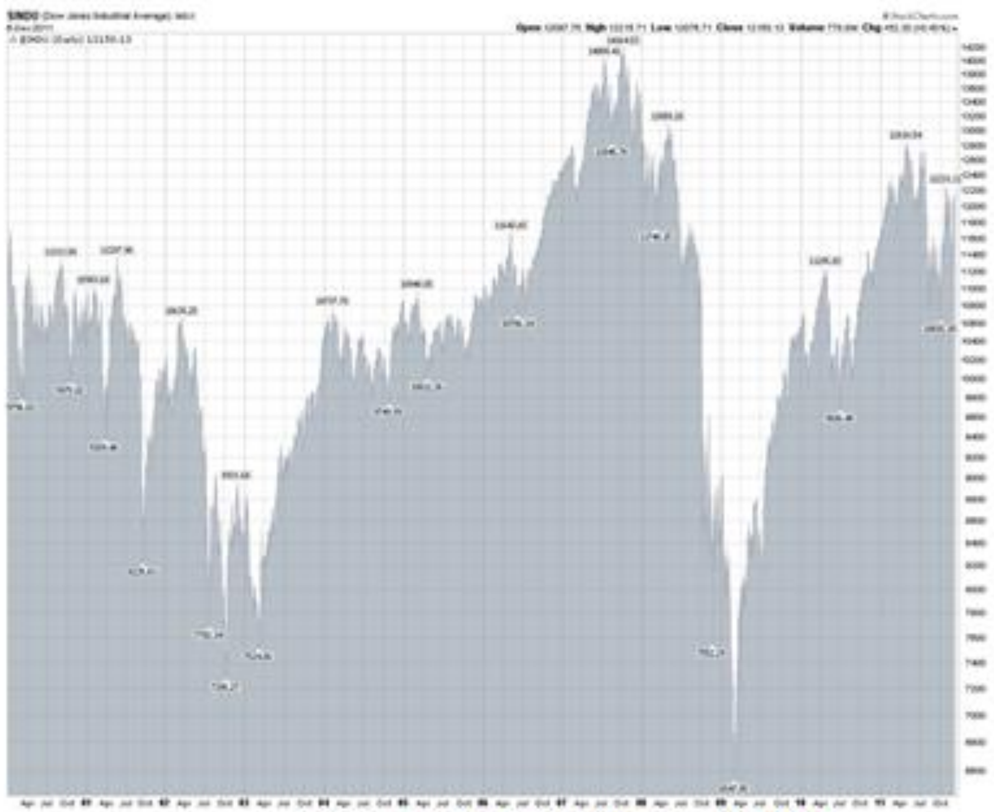


Housing Prices in US, Case-Shiller Index, 1987-2011

Figure 2: US Urban Housing Prices, 1987-2011

Source: Rosser, Rosser, and Gallegati (2012), Figure 2

The third case is historically the most common as documented by Kindleberger (1978, Appendix B). It involves prices rising to a peak, then declining for awhile in a gradual way during a “period of financial distress" (Minsky, 1972), then at a later time experiencing the Minsky Moment and crashing hard. During the financial crisis most financial asset markets showed this pattern, with Figure 3 showing the example of the US stock market as measured by the Dow-Jones Industrial Average. It peaked in October, 2007, but then crashed in September,2008, a full 11 months later after going through a period of a more erratic decline.

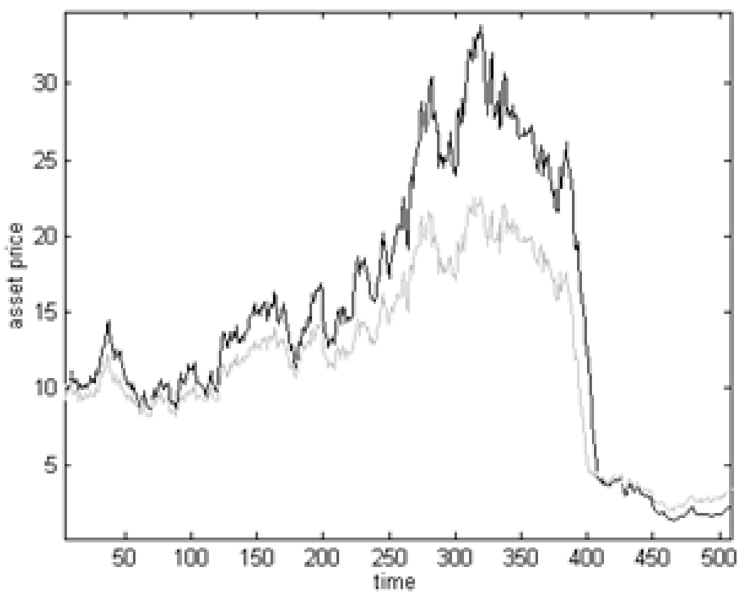


: Dow-Jones Industrial Average, 200-2011,
Daily Data

Figure 3: Daily Dow-Jones Industrial Average, 2000-2011

Source: Rosser, Rosser, and Gallegati (2012), Figure 3

Such dynamics cannot be modeled by assuming homogeneous agents. At the peak, smart or lucky "insiders" sell out to less smart or lucky "outsiders" who continue to hang on to the asset, much as the homeowners in the second case refuse to sell their homes initially as the price declines. The Minsky Moment finally arrives when panic hits this group of agents and they sell en masse in the crash. Even though this is by far the most common pattern of speculative bubbles in history, there have been few efforts to model this. Such an effort was made by Gallegati et al. (2011) in an agent-based model ultimately derived from the Brock-Hommes approach (Brock and Hommes, 1997). In this framework behavior of heterogeneous agents is qualitatively determined by a contagion parameter and a willingness to change behavior parameter. The mechanism of the delayed crash after the period of financial distress came from a wealth constraint such as agents encounter in asset markets with margin calls. When price falls below a certain level they may be forced to sell. Figure 4 shows a simulation b Gallegati et al (2011) that shows the general pattern and also shows the impact of an increase in the strength of the contagion parameter, which moves the peak higher and delays it slightly.



Increase in J. The two time series share the same random numbers and same parameters but J. In the grey time series J = 0.5; in the black time series J = 3.

Figure 4: Agent-Based Simulation of Minsky Bubble Process, Impact of Loosening Contagion Coefficient

Source: Gallegati, Palestrini, and Rosser (2011), Figure 10

Modeling Wealth and Income Distribution Dynamics Using Statistical Mechanics

Studying wealth and income distribution dynamics we find that the relationship between entropy-based non-power law distributions and power law distributions plays a central role in the modeling of these dynamical systems. In particular it increasingly looks as if while wealth dynamics largely reflect power law distributions, income distribution dynamics may be a combination, with entropy-related Boltzmann-Gibbs distributions best explaining income distribution for the poorest 97-98 percent, whereas a Pareto power law distribution may do better for the top level of income, where wealth dynamics may play a more important role (Dragulescu and Yakovenko, 2001; Yakovenko and Rosser, 2009).

Awareness of the possibility of using entropy ideas in the measurement of income distribution began with economists looking for generalizations of the various competing measures that have been used for studying income distributions. Thus in 1981, Cowell and Kuga (1981) sought a generalized axiomatic formulation for additive measures of income distribution. They found that by adding two axioms to the usual approach they were able to show that a generalized entropy approach could subsume the widely studied Atkinson measure (1970) and Theil measure (Bourguignon, 1979). While the Atkinson measure has been more widely used and is able to distinguish skewness of tails, the Theil may have more generality. Bourgignon (1979) shows that it is the only decomposable "income-weighted" inequality measure that is zero homogeneous. Cowell and Kuga (1981) show that adding a sensitivity axiom to their others yields the Theil index as the only one that is derivable from a generalized entropy concept.

These early discussions also involved strong claims regarding the difficulties of linking entropy measures with power law distributions, claims that now look to be overdone to some extent. Thus we find Montroll and Schlesinger (1983, p. 209) claiming that:

> "The derivation of distributions with inverse power tails from maximum entropy formalism would be a consequence only of an unconventional auxiliary condition that involves the specification of the average of a complicated logarithmic function."

This statement may be overdone, although indeed logarithmic functions are involved in the relationship between the two, which is not surprising given that entropy measures are essentially logarithmic.

The power law distribution approach dominates discussion of wealth distribution dynamics, as it does financial market dynamics. The father of this approach was Vilfredo Pareto (1897), who was initially trained as an engineer, but then became a socio-economist as his theory involved the relationship between social classes over time. Very appropriately Pareto's original motivation and focus of study was in fact income distribution. He claimed a universal truth associated with an estimated income distribution parameter. He was wrong, especially given that his theory fits better wealth distributions rather than income distributions, where, as noted above. Pareto argued incorrectly that his supposedly universal coefficient for the power law explanation of income distribution, which fit into his theory of the "circulation of the elites," in which nothing could be done to equalize income because the political process would simply involve substituting one power elite for another with no noticeable change in the income distribution. But we must recognize that he formulated this view at the end of the 19th century, when there had been a century of no major changes in the socio-economic structure anywhere. Needless to say, not too long afterwards there were large changes in the distribution, even as his method went “underground,” only to be revived for other uses such as describing urban metropolitan size distributions (Auerbach, 1913).

The modern concern with income distribution based on power law physics concepts from Pareto was due to a sociologist, John Angle (1986). After the appearance of current econophysics, many stepped forward to apply power law distributions to study the dynamics of wealth distributions. Drawing on the work of Pareto, who mistakenly thought he had found a universal coefficient for income distribution, econophysicists found that current wealth distributions fit Pareto’s power law view (Bouchaud and Mézard, 2000; Chakraborti and Chakraborti, 2000).

At this point the question needs to be considered as to whether we are dealing with ontological as opposed to “merely” metaphorical models in these matters. We know that there are stochastic tendencies for wealth and income dynamics, but it is not at all obvious that the various apparent imperatives for entropy maximization or minimization are actually driving outcomes. Nevertheless many studying these matters see thermodynamical processes underlying basic tendencies of wealth and income distribution dynamics. Such processes are not quite as direct as the ontological direction based on Carnot’s steam engines, but derive from broader tendencies of wealth and income distribution dynamics occurring in the absence of substantial changes in public policy regarding distributional policies.

Pareto was mistaken in his original proposal. He thought that he had found a universal law of income distribution that fit with his theory of the “circulation of the elites,” within which it did not matter which elite group was ruling society, the underlying distribution of income would not change. He was wrong. The legacy of his approach has been in the study of wealth distributions, where his presentation of power laws is now understood to explain wealth distributions rather than income distributions.

The Pareto distribution is given by:

$$N = A/x^{\alpha}, \tag{12}$$

where N is the number of observations above x, and A and α are constants. This includes as special cases a wide variety of other forms that underlie many econophysics models. The special case when $\alpha = 1$ leads to "Zipf's Law," (Zipf, 1941), widely viewed to describe urban size distributions as well as many others, although how far this "law" applies is a matter of ongoing debate.

Yakovenko and Rosser (2009) present a unified income distribution analysis combining an entropic Boltzmann-Gibbs formulation for lower income distribution with a Paretian power law distribution for the highest levels of income. The model makes a heroic assumption of conservation of money or income or wealth, which empirically is not unreasonable for the United States since the mid-1970s for median levels, even as the top strata have seen growing levels. But this fits with the combination of a lognormal entropic model for the majority of the population with regard to income, even as the top level of the income distribution seems to follow a wealth dynamic following a Paretian power law distribution.

Assuming a conservation of money, m, the entropically based Boltzmann-Gibbs equilibrium distribution is given by the probability, P, that the level will be m, given by:

$$P(\mathrm{m}) = \mathrm{c}e^{-m/T_m}, \tag{13}$$

where c is a normalizing constant, and T_m is the "money temperature" in thermodynamic terms, which is equal to the money supply per capita. This describes the lower portion of the income distribution.

Assuming a fixed rate of proportional money transfers with this equal to γ, the stationary distribution of money (income) is related to the Gamma distribution form that differs from the Boltzmann-Gibbs by having a power-law prefactor, m^{β},where

$$\beta = -1 - \ln 2/\ln(1 - \gamma). \quad (14)$$

This relates the Boltzmann-Gibbs form to a power law equivalent more simply than supposed by Montrell and Schlesinger [61]. This formulation that shows the connection between the two conceptualizations of wealth and income distributions is given by:

$$P(m) = cm^{\beta}e^{-m/T}. \quad (15)$$

This represents the stationary distribution, but allowing *m* to grow stochastically disconnects the outcome from the maximum entropy solution (Huang, 2004). The stationary distribution under these conditions becomes a mean-field case governed by a Fokker-Planck equation, which is neither Boltzmann-Gibbs nor Gamma, but is a version of a generalized Lotka-Volterra distribution, with *w* the wealth per person, *J* is the average transfer between agents, and σ is the standard deviation, and is

$$P(w) = c[(e^{-J/\sigma\sigma w})/(w^{2+J/\sigma\sigma})]. \quad (16)$$

So it is possible to combine an entropic Boltzmann-Gibbs formulation for the lower part of the income distribution with a power law form for its upper end, which corresponds to the wealth dynamics formulation deriving ironically from Pareto, given that he originally thought his conceptualization was a universal law of income distribution. His formulation would be countered soon after by Bachelier (1900), but we now see the two conjoined to provide an empirical explanation of income distribution that has deep roots in Marxist and other classical economic formulations regarding socio-economic class dynamics (Cockshott et al., 2009; Shaikh, 2016).

Figure 5 shows such a distribution in its log-log form for the US income distribution in 1997, with the Boltzmann-Gibbs portion, covering the lower 97 percent of the income distribution being nonlinear on the left-hand side, while the Paretian portion is linear in logs on the right-hand side covering the top 3 percent of the income distribution.

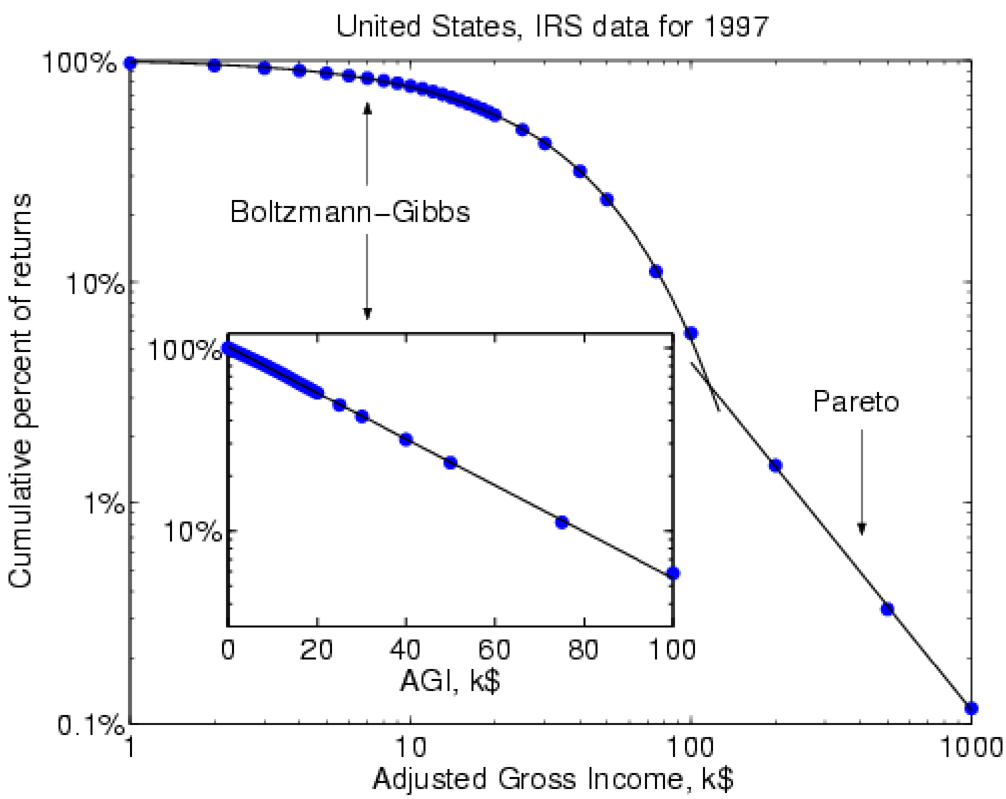


Figure 5: Log-log United States Income Distribution, Boltzmann-Gibbs and Pareto Sections, 1997

Source: Yakovenko with et al. (2011),Figure 5

Crashing Bubbles and the Revenge of Entropy

We now consider more specifically how the financial market dynamics interact with the income and wealth distribution dynamics in the course of speculative bubbles following a Minsky process. A notable aspect of a major bubble is that it raises the wealth and income of the top portion of the income and wealth distribution hierarchy compared to the rest. This is associated with the anti-entropic dynamics of the process and is reversed when the bubble disappears in a crash, the "revenge of entropy." This should show up during a bubble as an upward movement of the Paretian portion that will also move its boundary with the Boltzmann-Gibbs portion of the distribution to the left.

We do not have the data for the most recent financial crisis, nor do we have it for the Great Depression, another period that followed a major financial crash that has been posited to have sharply reduced wealth and somewhat equalized the income distribution, although wealth levels did decline substantially, the Great Depression bringing about the end of the "Gilded Age" (Smeeding, 2012). Events during the 2007-2009 Great Recession are more complicated in part because several different bubbles were involved, with the crash of the housing bubble heavily impacting the middle class while the stock market and derivatives market crashes more heavily affected the wealthy. Thus at its bottom point in 2009, the US stock market had fallen by more than half its value. Total wealth declined by the end of 2009 by 50 percent. Of this, wealth for the top 10 percent fell by 13 percent while the wealth of the top 1 percent fell by 20 percent (Smeeding, 2012). However, the stock market recovered rather rapidly, more so than in the 1930s or even after 2000, whereas the US housing market recovered much more slowly, thus leading to an outcome where while wealth inequality was probably reduced for a period of time during 2008-2009, it almost certainly rose after that as those at the top gained from the recovery of the stock market while those in the middle were held back by the continuing problems in the US housing market. The Minsky process was at work, but in a more complicated manner than in some other historical situations.

However, supporting evidence, if weak, can be seen from considering the end of the dotcom bubble in 2000. This can be seen in Figure 6 (Yakovenko, with et al [2011]), which shows the log-log relation for US income distribution for the years 1983-2001. In general, one sees little movement of the Boltzmann-Gibbs portion, but small annual changes of the Paretian part, reflecting steadily increasing inequality over time. However, there is one exception in this figure, what happened between 2000 and 2001, the last years shown, with 2000 the end of the dotcom bubble. In this case we see a reversal, with the 2001 Paretian portion lying below the 2000 portions. This would be consistent with our story of a revenge of entropy following the crash of the fairly substantial dotcom bubble of the late 1990s.

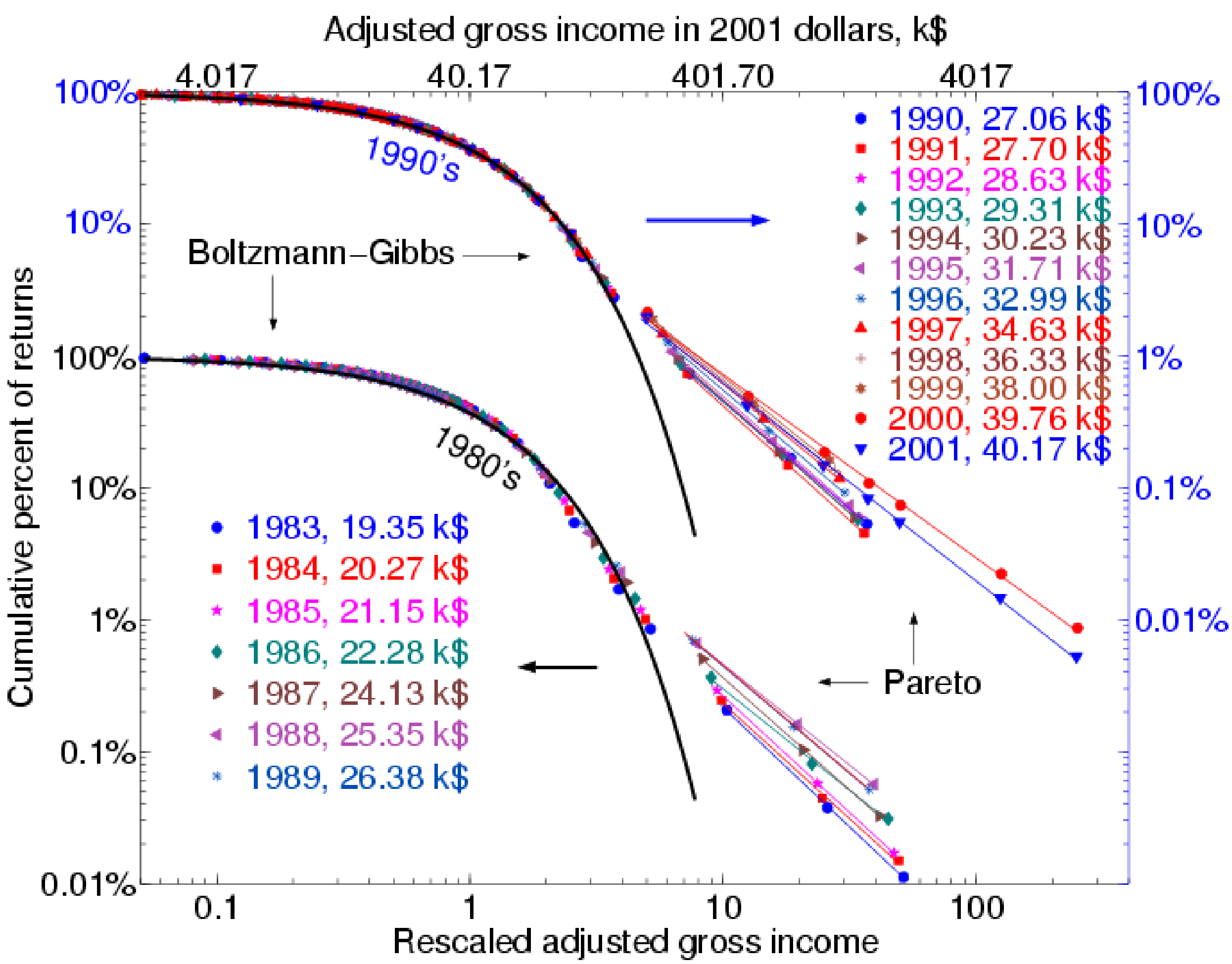


Figure 6: Annual log-log US Income Distribution, 1983-2001

Source: Yakovenko et al. (2011), Figure 6

Conclusions

We have seen that the Second Law of Thermodynamics, or Law of Entropy, plays a major role in the economy ontologically as a fundamental driving force through the dissipation of solar energy through the global ecosystem and economy, as well as more narrowly through industrial processes based on steam power. We have also seen that it can be used in a metaphorical way to mathematically model a variety of processes within the economy, from general equilibrium through financial market dynamics to income and wealth distribution dynamics. It provides a broad method for considering a stochastic perspective on economic dynamics and outcomes.

In the modeling of financial markets we see a conflict between entropic Gaussian stochastic equilibria that do not involve speculative bubble dynamics and an anti-entropic Minsky process wherein speculative bubbles emerge and eventually crash, although sometimes they merely gradually go back down as with the US housing bubble during the first decade of the twenty first century. The bubbles lead to kurtotic fat tailed outcomes in financial market returns as self-feeding positive feedback effects from momentum or noise traders lead to more extreme outcomes than are posited in entropic Gaussian situations. Bubbles drive markets out of entropic states into anti-entropic states that eventually come to an end as the bubble end and the markets return to longer run equilibria, the "revenge of entropy."

This can manifest itself in changes in income and wealth distributions, with income distributions able to be modeled by combining an entropic Boltzmann-Gibbs portion for the lower portion of the income distribution where income is largely earned from labor efforts, with a Paretian power law portion for the upper end of the income distribution where income is more likely to come from capital sources that are related to financial markets. Hence, financial market dynamics can lead to shifts of these portions relative to each other as the Paretian part expands with the rising wealth at the top end of the distribution during the rising part of a major speculative bubble, and then declining as the bubble ends with the revenge of entropy and a rising portion for the entropic Boltzmann-Gibbs portion of the distribution. We see possible evidence for such a dynamic in such a movement within the US income distribution following the end of the dotcom bubble in 2000.

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