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SOME REMARKS ON MELNIKOV CHAOS FOR SMOOTH AND PIECEWISE SMOOTH SYSTEMS.

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Abstract

It is well known that an autonomous system which has a homoclinic trajectory (i.e. a trajectory converging to a critical point as $t \to \pm\infty$) and subject to a small periodic forcing may exhibit a chaotic pattern. A motivating example in this context is given by a forced inverted pendulum.

Melnikov theory provides a computable sufficient condition for the existence of a transversal intersection between stable and unstable manifolds: in a smooth context this is enough to guarantee the persistence of the homoclinic and the insurgence of chaos.

In this talk we will briefly review these facts and we will show that, in a piecewise smooth context the situation is more complex: a geometrical obstruction may forbid chaotic phenomena which are replaced by new bifurcation scenarios. Further, if this obstruction is removed, chaos may arise again. Piecewise smooth system are motivated by the study of dry friction, state dependent switches, or impacts.

In fact we will also show some results new in a smooth context, concerning multiplicity, position and size of the Cantor set Σ of initial conditions from which chaos emanates. In particular we will see that, even if the perturbation is $O(\varepsilon)$, we may find infinitely many distinct Cantor set Σ located in the same $O(\varepsilon^\nu)$ neighborhood of the critical point, each corresponding to a different pattern, and where $\nu > 1$ is as large as we wish.

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