Concavity properties for solutions of the Logarithmic Schrödinger equation

Sunra Mosconi
University of Catania

I will discuss the existence of a solution to the logarithmic Schrödinger equation :

$$\begin{cases} -\Delta u = u \log u^2 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

in a bounded convex domain $\Omega \subseteq \mathbb{R}^N$, with the property that $\log u$ is concave.

Since the reaction $f(u) = u \log u^2$ is sign-changing and non-monotone, the classical techniques of [1, 3] to attack the problem fail. We instead rely on a continuity argument for the approximating Lane-Emden problems

$$\begin{cases} -\Delta u = \dfrac{2}{q-1}\,(u^q - u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

based on the heuristic

$$\frac{2}{q-1}\,(u^q - u) \quad \longrightarrow \quad u \log u^2 \qquad \text{as } q \to 1^+.$$

We will discuss the optimality of the result, exhibiting for any $\alpha > 0$ a solution of (1) such that u^α is *not* concave.

This is a joint work with M. Squassina and M. Gallo (Catholic University of Brescia), developed in the preprint [2].

References

[1] O. Alvarez, J.-M. Lasry, and P.-L. Lions, *Convex viscosity solutions and state constraints*, J. Math. Pures Appl. 76 (1997), 265–288.

[2] M. Gallo, S. Mosconi, M. Squassina, *Power law convergence and concavity for the logarithmic Schrödinger equation*, preprint arXiv:2411.01614.

[3] A.U. Kennington, *Power concavity and boundary value problems*, Indiana Univ. Math. J. 34 (1985), 687–704.