

A NOTE ON THE STOCHASTIC WEAKLY* ALMOST PERIODIC HOMOGENIZATION OF FULLY NONLINEAR ELLIPTIC EQUATIONS

HERMANO FRID

ABSTRACT. A function $f \in \text{BUC}(\mathbb{R}^d)$ is said to be weakly* almost periodic, denoted $f \in \mathcal{W}^*\text{AP}(\mathbb{R}^d)$, if there is $g \in \text{AP}(\mathbb{R}^d)$, such that, $M(|f - g|) = 0$, where $\text{BUC}(\mathbb{R}^d)$ and $\text{AP}(\mathbb{R}^d)$ are, respectively, the space of bounded uniformly continuous functions and the space of almost periodic functions, in \mathbb{R}^d , and $M(h)$ denotes the mean value of h , if it exists. We give a very simple direct proof of the stochastic homogenization property of the Dirichlet problem for fully nonlinear uniformly elliptic equations of the form $F(\omega, \frac{x}{\varepsilon}, D^2u) = 0$, $x \in U$, in a bounded domain $U \subseteq \mathbb{R}^d$, in the case where for almost all $\omega \in \Omega$, the realization $F(\omega, \cdot, M)$ is a weakly* almost periodic function, for all $M \in \mathcal{S}^d$, where \mathcal{S}^d is the space of $d \times d$ symmetric matrices. Here $(\Omega, \mu, \mathcal{F})$ is a probability space with probability measure μ and σ -algebra \mathcal{F} of μ -measurable subsets of Ω . For each fixed $M \in \mathcal{S}^d$, $F(\omega, y, M)$ is a stationary process, that is, $F(\omega, y, M) = \tilde{F}(T(y)\omega, M) := F(T(y)\omega, 0, M)$, where $T(y) : \Omega \rightarrow \Omega$ is an ergodic group of measure preserving mappings such that the mapping $(\omega, y) \rightarrow T(y)\omega$ is measurable. Also, $F(\omega, y, M)$, $M \in \mathcal{S}^d$, is uniformly elliptic, with ellipticity constants $0 < \lambda < \Lambda$ independent of $(\omega, y) \in \Omega \times \mathbb{R}^d$. The result presented here is a particular instance of the general theorem of Caffarelli, Souganidis and Wang, in CPAM 2005. Our point here is just to show a straightforward proof for this special case, which serves as a motivation for that general theorem, whose proof involves much more intricate arguments. We remark that any continuous stationary process verifies the property that almost all realizations belong to an ergodic algebra, and that $\mathcal{W}^*\text{AP}(\mathbb{R}^d)$ is, so far, the greatest known ergodic algebra on \mathbb{R}^d .

INSTITUTO DE MATEMÁTICA PURA E APLICADA - IMPA, ESTRADA DONA CASTORINA, 110, RIO DE JANEIRO, RJ, 22460-320,
BRAZIL
E-mail address: hermano@impa.br

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