Auxiliary Space Preconditioners for Discontinuous Galerkin Discretizations of H(curl)-elliptic Problems with Discontinuous Coefficients

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We propose a family of preconditioners for linear systems of equations ensuing from a piecewise polynomial symmetric Interior Penalty Discontinuous Galerkin (DG) discretization of $\mathrm{H}(\mathbf{curl}, \Omega)$-elliptic boundary value problems on conforming meshes arising in eddy current models. The design and analysis of the proposed solvers relies on the auxiliary space method (ASM): The preconditioners are obtained for the non-conforming DG approximation by using an auxiliary space of $\mathrm{H}(\mathbf{curl}, \Omega)$-conforming finite elements together with a relaxation technique (local smoothing).

On simplicial meshes, the proposed preconditioners enjoy asymptotic optimality with respect to mesh refinement. Moreover, we address the influence on the asymptotic performance of the preconditioners of possible discontinuities in the coefficients ν and β in the second and zero-th order parts of the operator respectively. Asymptotic optimality and robustness with respect to jumps in the coefficients can be shown, except when both coefficients are discontinuous and the problem is curl-dominated in some regions and reaction dominated in others. We validate the theory with extensive 2D numerical experiments.

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