$\mathbb{K}$ Körper

V Vektorraum über $\mathbb{K}$

$\{v_1, \dots, v_n\}$ $\mathbb{K}$-Basis von V

V, W $\quad \mathrm{Hom}_{\mathbb{K}}(V, W) = \{ f : V \to W \ \mathbb{K}\text{-linear} \}$

$\{w_1, \dots, w_m\}$ $\mathbb{K}$-Basis von W

$$\mathrm{Hom}_{\mathbb{K}}(V, W) \simeq \mathrm{Mat}_{m,n}(\mathbb{K})$$

$$f \longleftrightarrow (a_{ij})_{\substack{1 \le i \le m \\ 1 \le j \le n}}$$

$$f(v_j) = \sum_{i=1}^{m} a_{ij} w_i$$

$V^\vee = \mathrm{Hom}_{\mathbb{K}}(V, \mathbb{K})$

$\{v_1, \dots, v_n\}$ $\mathbb{K}$-Basis von V

$$V^\vee \simeq \mathrm{Mat}_{1,n}(\mathbb{K}) \simeq \mathbb{K}^n$$

$$\varphi \longleftrightarrow (\varphi(v_1), \dots, \varphi(v_n)) \longleftrightarrow \begin{pmatrix} \varphi(v_1) \\ \vdots \\ \varphi(v_n) \end{pmatrix}$$

we have a basis on $V^\vee$: $\{\varphi_1, \dots, \varphi_n\}$

so called dual basis of $\{v_1, \dots, v_n\}$

$$\varphi_i(v_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j . \end{cases}$$

$\mathbb{K}^n$ $e_1, \dots, e_n$ standard basis

$(\mathbb{K}^n)^\vee$ $dx_i : \mathbb{K}^n \longrightarrow \mathbb{K}$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \longmapsto y_i$$

$\{dx_1, \dots, dx_n\}$ dual basis

$$\omega : U \longrightarrow (\mathbb{R}^n)^\vee \simeq \mathbb{R}^n$$

$$\omega = \sum_{i=1}^{n} \omega_i \, dx_i$$

ω C^r $\iff$ $\omega_1, \dots, \omega_n$ are C^r

$$\omega_i : U \longrightarrow \mathbb{R}$$

rigorously: $X \subseteq \mathbb{R}^n$

$x_0 \in X$

$\{x_0\}$ Deformationsretrakt von X $\Rightarrow$ X zusammenziehbar

$\Downarrow$

$\pi_1(X, x_0)$ trivial

$i: S^1 \hookrightarrow \mathbb{C}^*$ Deformationsretrakt $\Rightarrow$ $\pi_1(i,1): \pi_1(S^1,1) \to \pi_1(\mathbb{C}^*,1)$ Gruppenisomorphismus

rigorous

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not rigorous but you can use these facts

- $\pi_1(S^1, 1) \simeq \mathbb{Z}$
- $\pi_1(\mathbb{C} \setminus \{p,q\})$ freie Gruppe mit 2 Erzeugern

$\gamma : [a,b] \longrightarrow U$ stückweise C^1, ω 1-Form auf U

$$\omega = \sum_{i=1}^{n} \omega_i \, dx_i \qquad \omega : U \longrightarrow (\mathbb{R}^n)^\vee$$

$$\gamma'(t) = \dot{\gamma}(t) \in \mathbb{R}^n \qquad \forall t \in [a,b] \setminus \left\{\begin{matrix}\text{finite}\\ \text{number}\\ \text{of points}\end{matrix}\right\}$$

$$\omega(\gamma(t)) \in (\mathbb{R}^n)^\vee$$

$$\omega(\gamma(t)) \, (\dot{\gamma}(t)) \in \mathbb{R}$$

$$dx_i : \mathbb{R}^n \longrightarrow \mathbb{R} \qquad \dot{\gamma}(t) = \begin{pmatrix} \dot{\gamma}_1(t) \\ \vdots \\ \dot{\gamma}_n(t) \end{pmatrix}, \quad dx_i(\dot{\gamma}(t)) = \dot{\gamma}_i(t)$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \longmapsto y_i$$

$$\omega(\gamma(t)) = \sum_{i=1}^{n} \omega_i(\gamma(t)) \cdot dx_i$$

$$\omega(\gamma(t))\,(\dot\gamma(t)) = \sum_{i=1}^{n} \omega_i(\gamma(t))\ \dot\gamma_i(t)$$

$\forall t \in [a,b] \setminus$ finitely many points

$$\int_a^b \omega(\gamma(t))(\dot\gamma(t))\,dt = \int_a^b \sum_{i=1}^{n} \omega_i(\gamma(t))\ \dot\gamma_i(t)\,dt$$

IN Definition von $\int_\gamma \omega$

05.1_Formen2