Algebraic Geometry Workshop in Bologna
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https://www.dm.unibo.it/~andrea.petracci3/2023SeptBolognaWorkshop/

Abstracts of minicourses

Yasunari Nagai
Hilbert scheme of points on a surface and its degeneration
Hilbert scheme of zero dimensional subschemes on a (quasi-)projective surface gives an interesting construction of higher dimensional smooth (quasi-)projective varieties of even dimension. For example, if the surface is a K3 surface, the Hilbert scheme gives an example of higher dimensional irreducible symplectic projective manifold. In the first part of this mini course, I explain the basic properties of the Hilbert scheme of points only assuming Hartshorne (i.e. a basic knowledge of modern algebraic geometry). In the second part, I put emphasis on the degeneration of Hilbert schemes. The motivation comes from the study of the boundary behavior of the period map of irreducibel symplectic Kähler manifolds. I also explain an explicit construction of the degeneration of Hilbert schemes.

Taro Sano
Deformations of Fano and Calabi-Yau varieties
In the classification of algebraic varieties, we parametrize varieties with similarities, thus it is natural to consider families/deformations of varieties. In good cases, we can parametrize some varieties over a smooth base space and this makes the description of the moduli space of those varieties reasonable.
Fano varieties and Calabi-Yau varieties are fundamental objects in the classification. In the talks, I'll explain that, when varieties are Fano or Calabi-Yau, we have such smooth parameter spaces. In most parts, I'll concentrate on smooth Fano/Calabi-Yau varieties. If time permits, I'll also talk about more general cases.

Abstracts of research talks

Daniele Faenzi
Moduli spaces of bundles in low genus and degeneracy loci
Coble hypersurfaces enjoy very special properties related to abelian varieties and moduli of semistable bundles of rank r with trivial determinant on a curve C of genus g, notably when (g,r) equal to (2,3) or (3,2).
Using orbital degeneracy loci arising from Vinberg theta-groups and Hecke cycles, we describe moduli of semistable bundles with fixed odd determinant as subvarieties of Grassmannians, again when (g,r) equals (2,3) or (3,2).
The geometry of these loci and of their singularities parallels that of Coble hypersurfaces and is related to projective models of K3 surfaces of genus 13 and 19.
Joint work with Vladimiro Benedetti, Michele Bolognesi, Laurent Manivel.

Dario Faro

Gauss-Prym maps on Enriques surfaces

Let C be a complex projective algebraic curve and let L and M be two line bundles on C. One can associate L and M with some natural maps between spaces of global sections of certain sheaves on C. These are called Gaussian-Wahl maps. These maps have been classically studied in connection with extendability questions of curves on surfaces. In this talk I will focus on the case of Enriques surfaces, presenting some natural questions that arise in this situation.

Lucas Li Bassi

The Fano variety of lines on a cyclic cubic fourfold

Among Fano varieties of K3 type (or FK3 for short) the cubic four- fold stands out for its historical significance. Indeed, long before the terminology FK3 was born there were already examples of a relation between this famous cubic hypersurface and irreducible holomorphic symplectic (or IHS for short) manifolds. One method to associate a smooth cubic fourfold with an IHS manifold involves the Fano variety of lines on it. This is, as proven by Beauville-Donagi, an IHS manifold of type K3[2]. This relation becomes even more intriguing when considering mildly singular cubic fourfolds, e.g. cubic fourfolds Y that are triple covering of P4 branched over a singular cubic threefold. In this case we have that F(Y), the Fano variety of lines on Y, is birational to an IHS manifold of type K3[2]. This fact has been used by Boissière-Camere-Sarti and by me to study some compactification of the moduli spaces of irreducible holomorphic symplectic manifolds with an order three non-symplectic automorphism. In order to achieve this result the authors do not consider the rich geometry of F(Y).

I will present recent results obtained in collaboration with Samuel Boissière and Paola Comparin that explain how the geometry of F(Y) gives us a better understanding of the deep relation between cyclic cubic fourfolds and IHS manifolds of type K3[2] with a non-symplectic automorphism of order three.

Alex Massarenti

On the (uni)rationality problem for quadric bundles and hypersurfaces

A variety X over a field is unirational if there is a dominant rational map from a projective space to X. We will discuss the unirationality problem for quartic hypersurfaces and quadric bundles over a arbitrary field in the the perspective of the relation between unirationality and rational connectedness. We will prove unirationality of quadric bundles under certain positivity assumptions on their anti-canonical divisor. As a consequence we will get the unirationality of any smooth 4-fold quadric bundle over the projective plane, over an algebraically closed field, and with discriminant of degree at most 12.

Benedetta Piroddi

Symplectic action of groups of order four on K3^[2]-type manifolds

When a group of order four G (either Z/4Z or (Z/2Z)^2) acts symplectically on a K3^[2]-type manifold X, then its action is always standard, meaning that we can deform the pair (X,G) to a pair (S^[2],G), where S^[2] is the Hilbert square of a K3 surface with a symplectic action of G, and the action of G on S^[2] is naturally induced.

I will describe these two actions and construct for each one the general member of a projective family that admits it, starting from a family of K3 surfaces with a mixed (symplectic and non-symplectic) action of a group of order 4.

Time permitting, I will also talk about the induced involutions on the Nikulin orbifold which is obtained by partial resolution of the quotient X/i, where i is a symplectic involution normal in G.

Eleonora Romano

Recent results on Fano varieties

In this talk we present some recent results on complex smooth Fano varieties. To this end, we first recall an invariant introduced by Casagrande, called Lefschetz defect. We review the literature to deduce that all Fano manifolds with Lefschetz defect greater than three are well known. Then we focus on the case in which the Lefschetz defect is equal to three, by discussing a structure theorem for such varieties. As an application, we use this result to classify all Fano 4-folds with Lefschetz defect equal to three: there are 19 families, among which 14 are toric. This is a joint work with C. Casagrande and S. Secci.

Filippo Viviani

On the classification of fine compactified Jacobians of nodal curves

If a smooth curve degenerates to a nodal curve, what are the possible modular degenerations of the Jacobian? I will give a complete answer to this question, using some recent results of Pagani-Tommasi.