MATHEMATICAL METHODS M – PART 2

University of Bologna
Master's Degree Programs in Telecommunications Engineering and in Electronic Engineering
Academic Year 2025/26, 1st semester

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https://www.dm.unibo.it/~andrea.petracci3/2025MathMethodsM/

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Prerequisites

Before the beginning of the class, students should know

- linear algebra: vector spaces, solving linear systems, determining bases of vector subspaces, matrix operations, determinants, eigenvalues and eigenvectors, matrix diagonalizability;
- calculus: limits, series, partial derivatives, one-variable integrals.

Preliminary syllabus

- Some topics in Linear algebra: spectral theorem and matrix exponential.
- Basic notions of complex analysis: holomorphic functions, radius of convergence of a power series, Taylor and Laurent series expansions for holomorphic functions, the z-transform and its region of convergence, poles of the z-transform.
- Basic notions of graph theory: basic definitions, properties of the eigenvalues of the Laplacian matrix.

References

Lectures

(1) **26th September 2025.** Practical information about the course. Numbers: naturals, integers, rationals, reals. Euler's number.

Definition of Taylor series of a real C^∞ function defined over an interval in $\mathbb{R}$. Definition of a real-analytic function defined over an interval in $\mathbb{R}$. Taylor series of e^x at 0. Taylor series of $\sin x$ at 0.

Definition of ring and of field. Complex numbers: real part, imaginary part, norm (or absolute value), conjugate. The complex number form a field: every non-zero complex number has a multiplicative inverse.

Definition of a polynomial in one variable. Roots/Zeroes of polynomials. Examples. Fundamental theorem of algebra: every polynomial with complex coefficients and of degree n has exactly n zeroes in $\mathbb{C}$, if counted with multiplicity.

Exponential of a complex number. Exponential form of complex numbers (complex logarithm problem): given $w \in \mathbb{C}^*$ find $z \in \mathbb{C}$ such that $e^z = w$. Example: find $z \in \mathbb{C}$ such that $e^z = 1 + \sqrt{3}i$.

Roots problem: given $w \in \mathbb{C}^*$ and $n \in \mathbb{N}^+$, find $z \in \mathbb{C}$ such that $z^n = w$. Example: find $z \in \mathbb{C}$ such that $z^2 = 1 + \sqrt{3}i$.

Roots of unity. Definition of $\zeta_n = \exp(2\pi i/n)$. The n n-th roots of unity are $1, \zeta_n, \zeta_n^2, \dots, \zeta_n^{n-1}$ and are the vertices of a regular n-gon inscribed in the unit circle.

Problem Sheet 1 — Mathematical Methods M, Part 2
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Problem 1.1. Determine the Taylor expansion of the following real functions at the indicated points:

(1) $\cos x$ at 0
(2) $\sin x$ at $\frac{\pi}{2}$
(3) $\log(1+x)$ at 0
(4) $\frac{1}{1-x}$ at 0
(5) $\frac{1}{x}$ at 1
(6) $\frac{1}{x}$ at -1
(7) $\frac{x}{1-x-x^2}$ at 0 (hard!)

Problem 1.2. Write each of the following complex numbers in the form $x+iy$, with $x, y \in \mathbb{R}$:

(1) $(3+4i)^2$
(2) $(3+4i)^3$
(3) $(3+4i)^{-1}$
(4) $(\frac{1}{2}+i\frac{\sqrt{3}}{2})^7$
(5) $2e^{i\frac{5}{4}\pi}$

Problem 1.3. Write each of the following complex numbers in the form $re^{i\theta}$, with $r \in (0,+\infty)$ and $\theta \in [0, 2\pi)$:

(1) $1+i$
(2) $3i^7$
(3) $\frac{\sqrt{3}}{2}+i\frac{1}{2}$
(4) $(\frac{\sqrt{3}}{2}+\frac{i}{2})^{1000}$

Problem 1.4.

(1) Find the 3rd roots of -1.
(2) Find the 4th roots of -1.
(3) Find the square roots of $36i$.
(4) Find the 3rd roots of $-27i$.

Problem 1.5. Draw each of the following subsets of the complex plane

(1) $\{z \in \mathbb{C} \mid z^3 = -1\}$
(2) $\{z \in \mathbb{C} \mid z^4 = -1\}$
(3) $\{z \in \mathbb{C} \mid z^2 = 36i\}$
(4) $\{z \in \mathbb{C} \mid z^3 = -27i\}$
(5) $\{z \in \mathbb{C} \mid |z| = 1\}$
(6) $\{z \in \mathbb{C} \mid 1 < |z-1| < 2\}$
(7) $\{z \in \mathbb{C} \mid -1 < \operatorname{Re} z < 2\}$
(8) $\{z \in \mathbb{C} \mid \operatorname{Re} z + \operatorname{Im} z \leq 1\}$

Problem 1.6. Find all the complex roots (aka zeroes) of the following polynomials and compute their multiplicity. Which of them are real? Which of them are rational?

(1) t^3+1
(2) t^4+1
(3) t^2-36i
(4) t^3+27i
(5) t^2+9
(6) $(t^2+9)^3$
(7) t^5-t
(8) $t^{10}-2t^6+t^2$
(9) t^2+1
(10) t^4+1
(11) $t^2+(3+2i)t+8-6i$
(12) $t^4+(3+2i)t^2+8-6i$