Mathematical Methods M, Part 2
Master's Degree Programs in Telecommunications Engineering and in Electronic Engineering
University of Bologna, A.Y. 2025/26, 1st semester; Instructor: Andrea Petracci

Written exam: 24th December 2025 (mock)

Rules

- Time available: 120 minutes.
- Allowed materials: one single A4 sheet with handwritten personal notes, and/or a basic (and practically unnecessary) non-programmable calculator.
- Prohibited materials: any additional notes, books, graphic or programmable calculators, mobile phones, tablets, laptops, smartwatches, or any device capable of communicating with the outside world (including earbuds or headphones). Any such device, if brought to the exam, must be switched off and stored inside a closed bag or backpack placed away from the candidate's desk. Possession of any prohibited device during the exam will result in immediate expulsion from the exam and formal reporting.
- Access to the exam room requires showing a valid university ID card with photograph. The ID card must remain visible on the desk for the entire duration of the exam.
- You do not need to rewrite the text of the questions. You do not need to submit this instruction sheet.
- Scoring: The total number of points available is 35. If x is the number of points obtained, the final grade will be the minimum between x and 30.
- The results will be published on Almaesami. Students may review their graded exam at the Department of Mathematics. If you elect not to accept the grade, you must notify me by email strictly within two working days after the publication of the results. If no communication is received within two working days, the grade will be considered definitively and irrevocably accepted.

Problem 1. (2 points) Find $a, b \in \mathbb{R}$ such that $a + bi = (2 + 3i)^{-1}$.

Problem 2. (5 points) Find all $z \in \mathbb{C}$ such that $z^3 = 27i$.

Problem 3. (12 points) Consider the matrix

$$A = \begin{pmatrix} 2 & 0 & 10 \\ 0 & 2 & -20 \\ 0 & 0 & -3 \end{pmatrix}$$

with real coefficients.

(1) Determine the determinant of A.
(2) Determine the characteristic polynomial of A.
(3) Determine the eigenvalues of A.
(4) For each eigenvalue λ, determine a basis of the eigenspace E_λ of A with respect to the eigenvalue λ.
(5) Is A diagonalisable? If yes, determine an invertible matrix $M \in \mathrm{M}_3(\mathbb{R})$ and a diagonal matrix $D \in \mathrm{M}_3(\mathbb{R})$ such that $M^{-1}AM = D$.

Problem 4. (8 points) Consider the sequence $(a_n)_{n \geq 0}$ defined by

$$\begin{cases} a_0 = 2, \\ a_1 = 4, \\ a_{n+2} = 4a_{n+1} - 5a_n \quad \text{for every } n \geq 0. \end{cases}$$

(1) Find a closed formula for a_n, i.e. write a_n as a function of n.
(2) Determine the radius of convergence of the power series $\sum_{n \geq 0} a_n z^n$, i.e. $R = 1/(\limsup_{n \to +\infty} \sqrt[n]{|a_n|})$.
(3) Find two polynomials $p(z)$ and $q(z)$ such that for every $z \in \mathbb{C}$ with $|z| < R$ one has $\sum_{n=0}^{+\infty} a_n z^n = p(z)/q(z)$.

Problem 5. (8 points) Consider the holomorphic function $f \colon \mathbb{C} \smallsetminus \{0, 2i, -2i\} \to \mathbb{C}$ given by

$$f(z) = \frac{1 + 3z}{z(z^2 + 4)}.$$

Find the inverse Z-transform of f on the anulus $A := \{z \in \mathbb{C} \mid 0 < |z| < 2\}$, i.e. determine the sequence $(u_n)_{n \in \mathbb{Z}}$ of complex numbers such that $f(z) = \sum_{n \in \mathbb{Z}} u_n z^{-n}$ for every $z \in A$.