MATHEMATICAL METHODS M – PART 2

University of Bologna
Master's Degree Programs in Telecommunications Engineering and in Electronic Engineering
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https://www.dm.unibo.it/~andrea.petracci3/2026MathMethodsM/

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Prerequisites

Before the beginning of the class, students should know

- linear algebra: vector spaces, solving linear systems, determining bases of vector subspaces, matrix operations, determinants, eigenvalues and eigenvectors, matrix diagonalizability;
- calculus: limits, series, partial derivatives, one-variable integrals.

Preliminary syllabus

- Some topics in Linear algebra: spectral theorem and matrix exponential.
- Basic notions of complex analysis: holomorphic functions, radius of convergence of a power series, Taylor and Laurent series expansions for holomorphic functions, the z-transform and its region of convergence, poles of the z-transform.

References

[AK19] Howard Anton and Anton Kaul. *Elementary linear algebra*. Wiley, 2019. 12th edition.
[GVL13] Gene H. Golub and Charles F. Van Loan. *Matrix computations*. The Johns Hopkins University Press, 2013. Fourth edition.
[Kre11] Erwin Kreyszig. *Advanced engineering mathematics*. Wiley, 2011. 10th edition.
[LLM16] David C. Lay, Stephen R. Lay, and Judi J. McDonald. *Linear Algebra and its Applications*. Pearson, 2016. Fifth edition.

Lectures

(1) **15th September 2026.** Practical information about the course.

Numbers: naturals, integers, rationals, reals. Decimal expansion.

Complex numbers: real part, imaginary part. Sum and product of two complex numbers: definition and their properties. Norm (or absolute value) and conjugate: definition and their properties. Complex numbers can be seen as points on the plane. Every non-zero complex number has a multiplicative inverse. The geometric meaning of the sum of two complex numbers is the sum of vectors. [LLM16, Appendix B] [AK19, Appendix B] [Kre11, 13.1]

Review of the Euler number and of the (natural) logarithm. Review of sinus and cosinus of "famous" angles.

Exponential of a complex number: definition and their properties. Polar form of a non-zero complex number and definition of argument. How to pass from the polar form to the cartesian form, and viceversa. The geometric meaning of the product by a complex number is rotation and dilation. [Kre11, 13.2].

(2) **22nd September 2026.** Solution of Problem 1 of the written exam of 14/07/2026.

Real logarithm. Complex logarithm problem: given $z \in \mathbb{C}$ such that $z \neq 0$, find all $w \in \mathbb{C}$ such that $\mathrm{e}^w = z$. Example: find all $w \in \mathbb{C}$ such that $\mathrm{e}^w = \frac{\mathrm{e}^2}{2} - \frac{\sqrt{3}\mathrm{e}^2}{2}\mathrm{i}$.

Real roots. Complex n-th root problem: given $z \in \mathbb{C}$ such that $z \neq 0$, find all $w \in \mathbb{C}$ such that $w^n = z$. Example: find all $w \in \mathbb{C}$ such that $w^3 = -27\mathrm{i}$.

Roots of unity. Definition of the primitive n-th root of unity: $\zeta_n := \exp(\frac{2\pi}{n}\mathrm{i})$. The n-th roots of unity are $1, \zeta_n, \zeta_n^2, \dots, \zeta_n^{n-1}$ and are the vertices of a regular n-gon inscribed in the unit circle.

Solution of degree 2 equations in the complex field. Solution of Problem 2 in the written test of 12/01/2026.

Recap of basic matrix operations: product of a row and a column, sum of two matrices, multiplication by scalar, product of two matrices, their properties, zero and identity matrices, transpose. [LLM16, 2.1] [AK19, 1.3, 1.4] [Kre11, 7.1, 7.2] [GVL13, 1.1]

Some suggested exercises

Problem 1. Write each of the following complex numbers in the form $x + iy$, with $x, y \in \mathbb{R}$:

(1) $(3 + 4\mathrm{i})^2$
(2) $(3 + 4\mathrm{i})^3$
(3) $(3 + 4\mathrm{i})^{-1}$
(4) $(\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2})^7$
(5) $2e^{\mathrm{i}\frac{5}{4}\pi}$

Problem 2. Write each of the following complex numbers in the form $re^{\mathrm{i}\theta}$, with $r \in (0, +\infty)$ and $\theta \in [0, 2\pi)$:

(1) $1 + \mathrm{i}$
(2) $3\mathrm{i}^7$
(3) $\frac{\sqrt{3}}{2} + \mathrm{i}\frac{1}{2}$
(4) $(\frac{\sqrt{3}}{2} + \frac{\mathrm{i}}{2})^{1000}$

Problem 3. Draw each of the following subsets of the complex plane

(1) $\{z \in \mathbb{C} \mid z^3 = -1\}$
(2) $\{z \in \mathbb{C} \mid z^4 = -1\}$
(3) $\{z \in \mathbb{C} \mid z^2 = 36\mathrm{i}\}$
(4) $\{z \in \mathbb{C} \mid z^3 = -27\mathrm{i}\}$
(5) $\{z \in \mathbb{C} \mid |z| = 1\}$
(6) $\{z \in \mathbb{C} \mid 1 < |z - 1| < 2\}$
(7) $\{z \in \mathbb{C} \mid -1 < \operatorname{Re} z < 2\}$
(8) $\{z \in \mathbb{C} \mid \operatorname{Re} z + \operatorname{Im} z \leq 1\}$

Problem 4. Solve Problem 1 in each of the written test given in the last academic year `https://www.dm.unibo.it/~andrea.petracci3/2025MathMethodsM/`

Problem 5.

(1) Find the 3rd roots of -1.
(2) Find the 4th roots of -1.
(3) Find the square roots of $36\mathrm{i}$.
(4) Find the 3rd roots of $-27\mathrm{i}$.

Problem 6. Solve Problem 2 in each of the written test given in the last academic year `https://www.dm.unibo.it/~andrea.petracci3/2025MathMethodsM/`

Problem 7. Pick one of the suggested books on complex analysis for engineers and solve many exercises about complex numbers.

Problem 8. Pick two arbitrary 3×3 matrices A and B, with coefficients in $\mathbb{R}$, and compute $A + B$, AB, BA, $\det(A)$, $\det(B)$, $\det(A + B)$, $\det(AB)$, $\det(BA)$. Then check your computations with some software.